Quiz for Lecture 15

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section 22
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1. Use polar coordinates to compute the double integral

$$\iint_D xy\,dA$$

where $D = \{(x,y) \mid x^2+y^2 \le 1, x \geq 0, y \geq 0\}$.

$D = \{(r,\theta) \mid r = 0..1, \theta = 0..\frac{\pi}{2}\}$.

$x = r\cos\theta \quad y = r\sin\theta$. $dA = r\,dr\,d\theta$

$$\int_0^{\frac{\pi}{2}}\int_0^1 r^3\cos\theta\sin\theta\,dr\,d\theta$$

~~$\cos\theta$~~

$\sin 2\theta = 2\cos\theta\sin\theta$

$\cos\theta\sin\theta = \frac{1}{2}\sin 2\theta$

$$\int_0^{\frac{\pi}{2}}\int_0^1 r^3\cdot\frac{1}{2}\sin 2\theta\,dr\,d\theta$$

$= \frac{1}{2}\sin 2\theta \int_0^1 r^3\,dr$

$= \frac{1}{2}\sin 2\theta\left[\frac{1}{4}r^4\Big|_0^1\right]$

$= \frac{1}{2}\sin 2\theta\cdot\frac{1}{4}$

$= \frac{1}{8}\sin 2\theta$

$$\int_0^{\frac{\pi}{2}}\frac{1}{8}\sin 2\theta\,d\theta$$

$= \frac{1}{8}\cos 2\theta\cdot\frac{1}{2}\Big|_0^{\frac{\pi}{2}}$

$= \frac{1}{4}[\cos\pi - \cos 0]$

$= \frac{1}{4}[-1-1]$

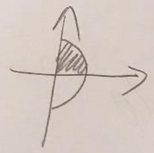
$= \frac{1}{4}\cdot -2$

$= -\frac{1}{2}$ Ans: $-\frac{1}{2}$

2、Evaluate the iterated integral by converting it to polar coordinates.

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2} \, \cancel{dy\,dx} \; \cancel{dx\,dy} \; dx\,dy$$

解 $D = \{(x,y) \mid x = 0..\sqrt{1-y^2},\ y = 0..1\}$.



$D = \{(r,\theta) \mid r = 0..1,\ \theta = 0..\frac{\pi}{2}\}$.

$$\int_0^{\frac{\pi}{2}} \int_0^1 e^{r^2} r \, dr \, d\theta .$$

① $\int_0^1 e^{r^2} r \, dr$

设 $u = r^2 \quad du = 2r\,dr$

$\frac{1}{2} du = r\,dr$

$= \int_0^1 e^u \frac{1}{2} du$

$= \frac{1}{2} e^u \Big|_0^1$

$= \frac{1}{2} e^{r^2} \Big|_0^1$

$= \frac{1}{2}(e^{1^2} - e^{0^2})$

$= \frac{1}{2}(e-1)$

$\frac{e-1}{2} \int_0^{\frac{\pi}{2}} d\theta$

$= \frac{e-1}{2} [\frac{\pi}{2} - 0]$

$= \frac{\pi(e-1)}{4}$.

Ans: $\frac{\pi(e-1)}{4}$.