

"QUIZ" for Lecture 14

NAME: (print!) SAI EMBAR Section: 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q14FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 26, 8:00pm

1. Evaluate the iterated integral

$$\int_0^1 \int_x^{3x} \int_0^y x^2 y z \, dz \, dy \, dx$$

$$\int_0^y x^2 y z \, dz = x^2 y \int_0^y z \, dz = x^2 y \left[\frac{z^2}{2} \right]_0^y = x^2 y \left(\frac{y^2}{2} - \frac{0^2}{2} \right) = \frac{x^2 y^3}{2}$$

$$\int_x^{3x} \frac{x^2 y^3}{2} \, dy = \frac{x^2}{2} \int_x^{3x} y^3 \, dy = \frac{x^2}{2} \left(\frac{y^4}{4} \Big|_x^{3x} \right) = \frac{x^2}{2} \left(\frac{(3x)^4 - x^4}{4} \right) =$$

$$\frac{x^2}{2} \left(\frac{81x^4 - x^4}{4} \right) = \frac{x^2}{2} \left(\frac{80x^4}{4} \right) = 10x^6$$

$$\int_0^1 10x^6 \, dx = 10 \frac{x^7}{7} \Big|_0^1 = \boxed{\frac{10}{7}}$$

2. Evaluate the triple integral

$$\iiint_E yz \ln(x^5) \, dV$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 2x \leq z \leq 3x\}$$

$$\int_0^1 \int_0^x \int_{2x}^{3x} yz \ln(x^5) \, dz \, dy \, dx$$

$$\int_{2x}^{3x} yz \ln(x^5) \, dz = y \ln(x^5) \int_{2x}^{3x} z \, dz$$

$$\int_{2x}^{3x} z \, dz = y \ln(x^5) \left[\frac{z^2}{2} \right]_{2x}^{3x} = y \ln(x^5) \frac{9x^2 - 4x^2}{2} = y \ln(x^5) \frac{5x^2}{2}$$

$$\int_0^x y \ln(x^5) \frac{5x^2}{2} \, dy = \ln(x^5) \frac{5x^2}{2} \int_0^x y \, dy = \ln(x^5) \frac{5x^2}{2} \left[\frac{y^2}{2} \right]_0^x = \ln(x^5) \frac{5x^2}{2} \left(\frac{x^2}{2} \right) = \ln(x^5) \frac{5x^4}{4}$$

$$\int_0^1 \ln(x^5) \frac{5x^4}{4} \, dx, \text{ do } u = x^5, \int_0^1 \ln(z) \frac{1}{4} dz = \frac{1}{4} \int_0^1 \ln(z) \, dz$$

$$\int \ln(z) \, dz = z \ln(z) - \int z (\ln(z))' \, dz = z \ln(z) - \int z \left(\frac{1}{z} \right) dz = z \ln(z) - z \, dz = z \ln(z) - z$$

$$\frac{1}{4} \int_0^1 \ln(z) \, dz = \frac{1}{4} (z \ln(z) - z) \Big|_0^1 = \frac{1}{4} (1 \ln(1) - 1) - (0) = \frac{1}{4} (1 \cdot 0 - 1) - (0) = \boxed{-\frac{1}{4}}$$