

Q14 Rahul Paleja

Section 22

$$\textcircled{1} \int_0^1 \int_x^{3x} \int_0^y x^2 y z \, dz \, dy \, dx$$

Inner Integral:

$$x^2 y \int_0^y z \, dz = \frac{z^2}{2} \Big|_0^y$$

$$x^2 y \cdot \frac{y^2}{2} = x^2 \cdot \frac{y^3}{2}$$

Middle:

$$\frac{x^2}{2} \int_x^{3x} y^3 \, dy = \frac{y^4}{4} \Big|_x^{3x}$$

$$= \frac{81x^4}{4} - \frac{x^4}{4} = \frac{80x^4}{4} = 20x^4$$

$$\frac{x^2}{2} \left(\frac{20x^4}{1} \right) = 10x^6$$

Outer:

$$\int_0^1 10x^6 \, dx = 10 \frac{x^7}{7} \Big|_0^1$$

$$= \boxed{\frac{10}{7}}$$

②

$$\iiint_E yz \ln(x^5) dV$$

$$= \int_0^1 \int_0^x \int_{2x}^{3x} yz \ln(x^5) dz dy dx$$

Inner:

$$y \ln(x^5) \int_{2x}^{3x} z dz = \frac{z^2}{2} \Big|_{2x}^{3x} = \frac{9x^2}{2} - \frac{4x^2}{2} = \frac{5x^2}{2} \cdot y \ln(x^5)$$

Middle:

$$\frac{5x^2}{2} \cdot \ln(x^5) \int_0^x y dy = \frac{x^2}{2} \cdot \frac{5x^2}{2} \ln(x^5) = \frac{5x^4}{2} \ln(x^5)$$

Outer:

$$\int_0^1 \ln(x^5) \frac{5x^4}{2} dx \quad u =$$

$$= \frac{1}{2} \int_0^1 \ln(x^5) \cdot 5x^4 dx$$

$$= \frac{1}{2} \int_0^1 25x^4 \ln(x) dx$$

$$= \frac{25}{2} \int_0^1 x^4 \cdot \ln(x) dx$$

$$\begin{aligned} u &= \ln(x) & v &= \frac{x^5}{5} \\ u' &= \frac{1}{x} dx & v' &= x^4 \\ \frac{x^5 \ln(x)}{5} - \frac{1}{5} \int x^4 dx \end{aligned}$$

$$\frac{25}{2} \left[\frac{x^5 \ln(x)}{5} - \frac{x^5}{25} \right]_0^1$$
$$= \frac{25}{2} \cdot \frac{-1^5}{25} = \boxed{\frac{-1}{2}}$$