Quiz for lecture 14.

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Section: 8:40-10:00 A.M.

1. Evaluate the iterated integral

$$\int_0^1 \int_x^{3x} \int_0^y x^2yz \, dz \, dy \, dx$$

$$\int_0^y x^2yz \, dz$$

$$= \left[\frac{x^2yz^2}{2}\right]_0^y$$

$$= \frac{x^2y^3}{2}$$

$$\int_x^{3x} \frac{x^2y^3}{2} \, dy$$

$$= \left[\frac{x^2y^4}{8}\right]_x^{3x}$$

$$= \frac{81}{8}x^6 - \frac{1}{8}x^6$$

$$= 10x^6$$

$$\int_0^1 10x^6 \, dx$$

$$= \left[\frac{10}{7}x^7\right]_0^1$$

$$= \frac{10}{7}$$

2. Evaluate the triple integral

$$\iiint_E yz\ln(x^5)\,dV$$

$$E=\{(x,y,z)\mid 0\le x\le 1,\ 0\le y\le x,\ 2x\le z\le 3x\}$$

$$\int_0^1\int_0^x\int_{2x}^{3x} yz\ln(x^5)\,dz\,dy\,dx.$$

$$\int_{2x}^{3x} yz\ln(x^5)\,dz$$

$$=\left[\ln(x^5)\,y\frac{z^2}{2}\right]_{2x}^{3x}$$

$$=\frac{5}{2}x^2\ln(x^5)\,y$$

$$\int_0^x \frac{5}{2}x^2\ln(x^5)\,y\,dy$$

$$=\left[\frac{5}{2}x^2\ln(x^5)\frac{y^2}{2}\right]_0^x$$

$$=\frac{5}{4}x^4\ln(x^5)$$

$$\int_0^1 \frac{5}{4}x^4\ln(x^5)\,dx$$

$$=\left[\frac{5}{4}x^5\ln x-\frac{1}{4}x^5\right]_0^1$$

$$=-\frac{1}{4}$$