

"QUIZ" for Lecture 14

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q14FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 26, 8:00pm

1. Evaluate the iterated integral

$$\int_0^1 \int_x^{3x} \int_0^y x^2 y z \, dz \, dy \, dx$$

$$\int_0^y x^2 y z \, dz = x^2 y \left. \frac{z^2}{2} \right|_0^y = x^2 y \frac{y^2}{2} = \frac{x^2 y^3}{2}$$

$$\int_x^{3x} \frac{x^2 y^3}{2} \, dy = \left. \frac{x^2 \cdot \frac{y^4}{4}}{2} \right|_x^{3x} = \left( \frac{x^2}{2} \cdot \frac{(3x)^4}{4} \right) - \left( \frac{x^2}{2} \cdot \frac{x^4}{4} \right)$$

$$= \frac{81x^6}{8} - \frac{x^6}{8} = 10x^6$$

$$\int_0^1 10x^6 \, dx = 10 \cdot \left. \frac{x^7}{7} \right|_0^1 = \boxed{\frac{10}{7}}$$

2. Evaluate the triple integral

$$\iiint_E yz \ln(x^5) \, dV$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 2x \leq z \leq 3x\}$$

IBP  
 $v = \ln x$   
 $u = \frac{1}{x}$   
 $v' = \frac{x^5}{5}$   
 $u' = x^4$

$$\int_0^1 \int_0^x \int_{2x}^{3x} yz \ln(x^5) \, dz \, dy \, dx$$

$$\int_{2x}^{3x} yz \ln(x^5) \, dz = y \ln(x^5) \left( \frac{z^2}{2} \right) \Big|_{2x}^{3x} = y \ln(x^5) \left( \frac{9x^2}{2} - \frac{4x^2}{2} \right) = y \ln(x^5) \left( \frac{5x^2}{2} \right)$$

$$\int_0^x y \ln(x^5) \left( \frac{5x^2}{2} \right) \, dy = \left[ \ln(x^5) \left( \frac{5x^2}{2} \right) \left( \frac{y^2}{2} \right) \right] \Big|_0^x = \frac{5 \ln(x^5) x^2}{2} \cdot \frac{x^2}{2}$$

$$\frac{25}{4} \int_0^1 (\ln(x^5)) x^4 \, dx = \ln x \cdot x^4 - \int_0^1 \frac{1}{x} \cdot \frac{x^6}{5} \, dx = \frac{25}{20} \left( \ln x \cdot x^4 - \frac{x^5}{5} \right) \Big|_0^1$$

$$= \frac{25}{20} \left( (\ln(1) - \frac{1}{5}) - (\ln(0) \cdot 0 - 0) \right) = \frac{25}{20} \left( -\frac{1}{5} \right) = \boxed{-\frac{1}{4}}$$