

"QUIZ" for Lecture 14

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q14FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 26, 8:00pm

1. Evaluate the iterated integral

$$\int_0^1 \int_x^{3x} \int_0^y x^2 y z \, dz \, dy \, dx$$

Inner integral:

$$\int_0^y x^2 y z \, dz = \frac{x^2 y z^2}{2} \Big|_0^y = \frac{x^2 y^3}{2}$$

Middle integral:

$$\int_x^{3x} \frac{x^2 y^3}{2} \, dy = \frac{x^2 y^4}{8} \Big|_x^{3x} = \frac{x^2 (3x)^4}{8} - \frac{x^2 x^4}{8} = \frac{(81-1)x^6}{8} = 10x^6$$

Outer integral:

$$\int_0^1 10x^6 \, dx = \frac{10x^7}{7} \Big|_0^1 = \boxed{\frac{10}{7}}$$

2. Evaluate the triple integral

$$\iiint_E yz \ln(x^5) \, dV$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 2x \leq z \leq 3x\}$$

Inner integral:

$$\int_{2x}^{3x} yz \ln(x^5) \, dz = \frac{yz^2}{2} \ln(x^5) \Big|_{2x}^{3x} = \left(\frac{9yx^2}{2} - \frac{4yx^2}{2} \right) \ln(x^5) = \frac{5}{2} yx^2 \ln(x^5)$$

Middle integral:

$$\int_0^x \frac{5}{2} yx^2 \ln(x^5) \, dy = \frac{5}{4} y^2 x^2 \ln(x^5) \Big|_0^x = \frac{5x^4}{4} \ln(x^5)$$

Outer integral:

$$\int_0^1 \frac{5x^4}{4} \ln(x^5) \, dx \rightarrow \text{substitute } u = x^5 \rightarrow du = 5x^4 \, dx \rightarrow \int_0^1 \frac{\ln(u)}{4} \, du = \frac{1}{4} (4 \ln(u) - u) \Big|_0^1 =$$

$$= \frac{1}{4}(1/4(1) - 1) = \frac{1}{4}(-1) = \boxed{-\frac{1}{4}}$$