

"QUIZ" for Lecture 14

NAME: (print!) Gillian Mulvey Section: _____

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q14FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 26, 8:00pm

1. Evaluate the iterated integral

$$\int_0^1 \int_x^{3x} \int_0^y x^2 y z \, dz \, dy \, dx$$

$$\int_0^y x^2 y z \, dz = x^2 y \frac{z^2}{2} \Big|_0^y = \frac{x^2 y^3}{2}$$

$$\int_x^{3x} \frac{x^2 y^3}{2} \, dy = \frac{x^2 y^4}{8} \Big|_x^{3x} = \frac{x^2}{8} \left(\frac{(3x)^4}{8} - \frac{x^4}{8} \right) = \frac{x^2}{8} \left(\frac{81x^4}{8} - \frac{x^4}{8} \right) = \frac{10x^6}{8} = \frac{5x^6}{4}$$

$$\int_0^1 \frac{5x^6}{4} \, dx = \frac{5}{4} \frac{x^7}{7} \Big|_0^1 = \frac{5}{4} \left(\frac{1^7}{7} - 0 \right) = \frac{5}{28}$$

2. Evaluate the triple integral

$$\iiint_E yz \ln(x^5) \, dV$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 2x \leq z \leq 3x\}$$

$$\int_0^1 \int_0^x \int_{2x}^{3x} yz \ln(x^5) \, dz \, dy \, dx$$

$$\int_{2x}^{3x} yz \ln(x^5) \, dz = \frac{z^2}{2} \cdot y \ln(x^5) \Big|_{2x}^{3x} = y \ln(x^5) \left(\frac{(3x)^2}{2} - \frac{(2x)^2}{2} \right) = y \ln(x^5) \frac{5x^2}{2}$$

$$\int_0^x y \ln(x^5) \frac{5x^2}{2} \, dy = \ln(x^5) \frac{5x^2}{2} \left(\frac{y^2}{2} - 0 \right) = \ln(x^5) \cdot \frac{5x^4}{4}$$

$$\int_0^1 \frac{5x^4}{4} \ln(x^5) \, dx = \frac{5}{4} \int_0^1 x^4 \ln(x^5) \, dx = \frac{5}{4} \left(\frac{x^5}{5} \ln(x^5) - \int x^4 \right)$$

$u = \ln(x^5) \quad du = x^4$
 $dv = \frac{1}{x^5} \cdot 5x^4 \quad v = \frac{x^5}{5}$

$$= \frac{5}{4} \left(\frac{x^5}{5} \ln(x^5) - \frac{x^5}{5} \right) \Big|_0^1 = \frac{5}{4} \left(\frac{1}{5} \ln(1) - \frac{1}{5} \right) = \frac{-1}{4} = -\frac{1}{4}$$