

"QUIZ" for Lecture 14

NAME: (print!) Afsana Rahman Section: 25

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q14FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 26, 8:00pm

1. Evaluate the iterated integral

$$\int_0^1 \int_x^{3x} \int_0^y x^2 y z \, dz \, dy \, dx$$

$$\int_0^1 \int_x^{3x} \int_0^y x^2 y z \, dz \, dy \, dx = \int_0^1 \int_x^{3x} (x^2 y) \left(\frac{z^2}{2} \right)_0^y \, dy \, dx = \int_0^1 \int_x^{3x} \frac{1}{2} x^2 y^2 \, dy \, dx$$

$$\int_0^1 \int_x^{3x} \frac{1}{2} x^2 y^2 \, dy \, dx = \int_0^1 \left(\frac{1}{2} x^2 \right) \left(\frac{1}{3} y^3 \right)_x^{3x} \, dx = \int_0^1 \frac{1}{2} x^2 \left(\frac{1}{3} (81x^3 - x^3) \right) \, dx = \int_0^1 10x^6 \, dx$$

$$\int_0^1 10x^6 \, dx = \frac{10}{7} x^7 \Big|_0^1 = \left(\frac{10}{7} \right)$$

2. Evaluate the triple integral

$$\iiint_E yz \ln(x^5) \, dV$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 2x \leq z \leq 3x\}$$

$$\int_0^1 \int_0^x \int_{2x}^{3x} yz \ln(x^5) \, dz \, dy \, dx = \int_0^1 \int_0^x \frac{1}{2} y z^2 \ln(x^5) \Big|_{2x}^{3x} \, dy \, dx = \int_0^1 \int_0^x (9x^2 - 4x^2) \frac{1}{2} y \ln x^5 \, dy \, dx$$

$$\int_0^1 \int_0^x \frac{5}{2} x^2 y \ln x^5 \, dy \, dx = \int_0^1 \frac{5}{4} y^2 x^2 \ln x^5 \Big|_0^x \, dx = \int_0^1 \frac{5}{4} x^4 \ln x^5 \, dx$$

$$\frac{5}{4} \int_0^1 x^4 \ln x^5 \, dx \quad \begin{matrix} u = x^5 \\ du = 5x^4 \end{matrix} \Rightarrow \frac{5}{16} \int_0^1 \ln u \, du = \frac{5}{16} (u \ln u - u) \Big|_0^1 = \frac{5}{16} (x^5 \ln x^5 - x^5) \Big|_0^1$$

$$\frac{5}{16} (\ln 1 - 1) = \left(-\frac{5}{16} \right)$$