

"QUIZ" for Lecture 14

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q14FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 26, 8:00pm

1. Evaluate the iterated integral

$$\int_0^1 \int_0^{3x} \int_0^y x^2 dz dy dx = \int_0^1 \int_x^{3x} z x^2 \Big|_0^y dy dx = \int_0^1 \int_x^{3x} y x^2 dy dx$$

$$= \int_0^1 \int_x^{3x} y x^2 dy dx = \int_0^1 \frac{x^2 y^2}{2} \Big|_x^{3x} dx = \int_0^1 \left(\frac{9x^4}{2} - \frac{x^4}{2} \right) dx$$

$$= \int_0^1 \frac{8x^4}{2} dx = \int_0^1 4x^4 dx = \frac{4x^5}{5} \Big|_0^1 = \boxed{\frac{4}{5}}$$

2. Evaluate the triple integral

$$\iiint_E yz \ln(x^5) dV,$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 2x \leq z \leq 3x\}$$

$$= \int_0^1 \int_0^x \int_{2x}^{3x} yz \ln(x^5) dz dy dx = \int_0^1 \int_0^x y \ln(x^5) \left[\frac{z^2}{2} \right]_{2x}^{3x} dy dx = \int_0^1 \int_0^x \frac{5x^2 y \ln(x^5)}{2} dy dx$$

$$\int_0^1 \int_0^x \frac{5x^2 y \ln(x^5)}{2} dy dx = \int_0^1 \frac{5x^2 \ln(x^5)}{2} \left[\frac{y^2}{2} \right]_0^x dx = \int_0^1 \frac{5x^4 \ln(x^5)}{4} dx$$

$$\int_0^1 \frac{5x^4 \ln(x^5)}{4} dx = \int_0^1 \frac{25x^4 \ln(x)}{4} dx \quad \begin{matrix} u = \ln(x) & du = \frac{1}{x} dx \\ dv = x^4 dx & v = x^5/5 \end{matrix}$$

$$\int u dv = uv - \int v du \quad \frac{25}{4} \left[\frac{\ln(x) x^5}{5} - \int \frac{x^4}{5} dx \right] = \frac{25}{4} \left[\frac{\ln(x) x^5}{5} - \frac{x^5}{25} \right] \Big|_0^1 = \boxed{-\frac{1}{4}}$$