

Q1. 14

$$Q1. \int_0^1 \int_x^{3x} \int_0^y (x^2 y z) dz dy dx$$

$$\begin{aligned} \Rightarrow \int_0^y x^2 y z \cdot dz &= x^2 y \cdot \left(\frac{1}{2} z^2\right) \Big|_0^y \\ &= x^2 y \cdot \frac{y^2}{2} \\ &= \frac{1}{2} x^2 y^3 \end{aligned} \quad \int_x^{3x} \frac{1}{2} x^2 y^3 dy = \frac{1}{2} x^2 \cdot \left(\frac{1}{4} y^4\right) \Big|_x^{3x}$$

$$= \frac{1}{2} x^2 \cdot \left(\frac{(3x)^4}{4} - \frac{x^4}{4}\right) = \frac{1}{2} x^2 \cdot \frac{80x^4}{4} = 10x^6$$

$$\int_0^1 10x^6 dx = 10 \cdot \frac{1}{7} x^7 \Big|_0^1 = 10 \cdot \frac{1}{7} = \frac{10}{7}$$

$$Q2. \iiint_E yz \ln(x^5) dV$$

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 2x \leq z \leq 3x\}$$

$$\int_0^1 \int_0^x \int_{2x}^{3x} yz (\ln x^5) dz dy dx$$

$$\int_{2x}^{3x} yz (\ln x^5) dz$$

$$= y (\ln x^5) \left(\frac{1}{2} z^2\right) \Big|_{2x}^{3x}$$

$$= y (\ln x^5) \cdot \left(\frac{1}{2} 9x^2 - \frac{1}{2} 4x^2\right)$$

$$= y (\ln x^5) \cdot \frac{5x^2}{2}$$

$$\int_0^x y (\ln x^5) \cdot \frac{5x^2}{2} dy$$

$$= \frac{5x^2 \cdot \ln(x^5)}{2} \int_0^x y dy$$

$$= \frac{5x^2 \cdot \ln(x^5)}{2} \cdot \frac{1}{2} y^2 \Big|_0^x$$

$$\int_0^1 \frac{5x^4 \cdot \ln x^5}{4} dx$$

$$= \frac{1}{4} \int_0^1 5x^4 \cdot \ln x^5 dx$$

$$= \frac{1}{4} \cdot (x^5 \ln x^5 - x^5) \Big|_0^1$$

$$= \frac{1}{4} \cdot ((\ln 1 - 1) - 0)$$

$$= -\frac{1}{4}$$

KOKUYO

