

Quiz for Lecture 14.

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Section 22

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1. Evaluate the iterated integral

$$\int_0^1 \int_x^{3x} \int_0^y x^2 y z \, dz dy dx$$

$$\textcircled{1} \int_0^y x^2 y \, dz$$

$$= x^2 y \left[\frac{1}{2} z^2 \Big|_0^y \right]$$

$$= x^2 y \cdot \frac{1}{2} y^2$$

$$= \frac{1}{2} x^2 y^3$$

$$\textcircled{2} \int_x^{3x} \frac{1}{2} x^2 y^3 \, dy$$

$$= \frac{1}{2} x^2 \left[\frac{1}{4} y^4 \Big|_x^{3x} \right]$$

$$= \frac{1}{2} x^2 \left[\frac{1}{4} (3x)^4 - \frac{1}{4} x^4 \right]$$

$$= \frac{1}{2} x^2 \left[\frac{81}{4} x^4 - \frac{1}{4} x^4 \right]$$

$$= \frac{1}{2} x^2 \cdot \frac{80}{4} x^4$$

$$= 10 x^6$$

$$\textcircled{3} \int_0^1 10 x^6 \, dx$$

$$= 10 \left[\frac{1}{7} x^7 \Big|_0^1 \right]$$

$$= 10 x \frac{1}{7}$$

$$= \frac{10}{7}$$

Ans: $\frac{10}{7}$

2. Evaluate the triple integral

$$\iiint_E yz \ln(x^5) dv$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, x \leq z \leq 3x\}$$

$$\begin{aligned} \textcircled{1} \quad & \int_0^1 \int_0^x \int_{x^2}^{3x} yz \ln(x^5) dz dy dx \\ & = \int_0^1 \int_0^x \frac{5}{2} x^2 y \ln(x^5) dy dx \\ & = \frac{5}{2} x^2 \ln(x^5) \left[\frac{1}{2} y^2 \right]_0^x \\ & = \frac{5}{2} x^2 \ln(x^5) \cdot \frac{1}{2} x^2 \\ & = \frac{5}{4} x^4 \ln(x^5) \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & \int_0^1 \frac{5}{4} x^4 \ln(x^5) dx \\ & u = x^5 \quad du = 5x^4 dx \\ & \frac{1}{5} du = x^4 dx \\ & \int \frac{1}{5} du \ln u \\ & = \frac{1}{5} [u \ln u - \int \ln u du] \\ & = \frac{1}{5} [x^5 \ln(x^5) - x^5] \Big|_0^1 \\ & = \frac{1}{5} [(1 \cdot \ln(1) - 1) - (0 - 0)] \\ & = \frac{1}{5} [-1] \\ & = -\frac{1}{5} \end{aligned}$$

$$\begin{aligned} \int u dv &= uv - \int v du \\ &= \frac{5}{24} x^6 \cdot \ln(x^5) - \int \frac{5}{24} x^5 \cdot \frac{5}{x} dx \\ &= \frac{5}{24} x^6 \ln(x^5) - \frac{25}{24} x^5 \Big|_0^1 \\ &= \frac{5}{24} \cdot 0 - \frac{25}{24} = -\frac{25}{24} \end{aligned}$$

ANS: $-\frac{1}{4}$