"QUIZ" for Lecture 13

NAME: (print!) SAI EMBAR **Section:** 23

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q13FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 22, 8:00pm

1. Change the order of integration in

$$\int_1^4 \int_0^{\ln y} f(x,y)\,dx\,dy \quad .$$

Region of integration = $\{(x,y) \mid 1 \le y \le 4, 0 \le x \le \ln y\}$

Horizontally simple, $(0,1) \to (0,4)$ it's one push

$x=0 \to x = \ln y$

$x=0 \to x = \ln 4$.

$x = \ln y \to y = e^x$ to link $y=4$.

$D = \{(x,y) \mid 0 \le x \le \ln 4, e^x \le y \le 4\}$

$\therefore \int_0^{\ln 4} \int_{e^x}^4 f(x,y)\,dy\,dx$

2. Evaluate

$$\int_0^2 \int_{y/2}^1 \frac{1}{(x^2+1)^2}\,dx\,dy \quad ,$$

by inverting the order of integration and evaluating the new iterated integral.

$D = \{(x,y) \mid 0 \le y \le 2, y/2 \le x \le 1\}$

$y=0 \to y=2$ main road

$x = y/2$ to $x=1$ side street.

projection on x-axis is from $x=0$ to $x=1$ and horizontal side street starts at $y=0$ to $x=y/2$.

$y = 2x$, $D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 2x\}$

$\int_0^1 \int_0^{2x} \frac{1}{(x^2+1)^2}\,dy\,dx$

$\int_0^{2x} \frac{1}{(x^2+1)^2}\,dy = \frac{1}{(x^2+1)^2}\left(\int_0^{2x} dy\right)$

$= \frac{1}{(x^2+1)^2}\, y\Big|_0^{2x} = \frac{1}{(x^2+1)^2}(2x-0) = \frac{2x}{(x^2+1)^2}$

$\int_0^1 \frac{2x}{(x^2+1)^2}\,dx$ $\quad u = x^2+1$, $2x\,dx = du$

$\int_1^2 \frac{1}{u^2}\,du = \frac{u^{-1}}{-1}\Big|_1^2$

$= \frac{-1}{u}\Big|_1^2 = \boxed{\frac{1}{2}}$