

quiz for Lecture 13

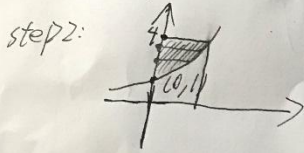
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section 22

10.23.2020

1. Change the order of integration in

$$\int_1^4 \int_0^{\ln y} f(x, y) dx dy$$

step 1:  $x=0$   $x=\ln y \rightarrow y=e^x$   
 $y=1, 4$



$$D = \{(x, y) \mid e^x \leq y \leq 4, 0 \leq x \leq \ln 4\}$$

step 3:

$$\int_0^{\ln 4} \int_{e^x}^4 f(x, y) dy dx$$

This is Ans.

2. Evaluate

$$\int_0^2 \int_{\frac{y}{2}}^1 \frac{1}{(x^2+1)^2} dx dy$$

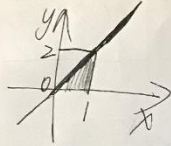
by inverting the order of integration and evaluating the new iterated integral.

step 1:  $\int_0^2 \int_{\frac{y}{2}}^1 f(x, y) dx dy$

min:  $x = \frac{y}{2} \rightarrow y = 2x$

max:  $x = 1$

step 2:



$$D = \{(x,y) \mid 0 \leq y \leq 2x, 0 \leq x \leq 1\}$$

step 3:

$$\int_0^1 \int_0^{2x} \frac{1}{(x^2+1)^2} dy dx$$

step 4:

$$\int_0^{2x} \frac{1}{(x^2+1)^2} dy$$

$$= \frac{1}{(x^2+1)^2} y \Big|_0^{2x}$$

$$= \frac{1}{(x^2+1)^2} 2x$$

step 5:  $\int_0^1 \frac{2x}{(x^2+1)^2} dx$

$$u = x^2 + 1 \quad du = 2x dx$$

$$\int \frac{1}{u^2} du$$

$$= -\frac{1}{u}$$

$$\int_0^1 \frac{1}{x^2+1} \Big|_0^1$$

$$= -\frac{1}{1^2+1} - \left(-\frac{1}{0^2+1}\right)$$

$$= -\frac{1}{2} + 1$$

$$= \frac{1}{2}$$

ANS:  $\frac{1}{2}$