10/18/2020

Yash Khanguria "Quiz" for lecture 11 Section 24

1. Use Lagrange Multipliers to find the smallest value that $x+y+z$ can be, given that $xyz = 125$

$\nabla f(x,y,z) = \langle 1,1,1 \rangle$ $\qquad$ $\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$

$\nabla g(x,y,z) = \langle yz, xz, xy \rangle$ $\qquad$ $\langle 1,1,1 \rangle = \lambda \langle yz, xz, xy \rangle$

$1 = \lambda yz$ $\qquad$ $1 = \lambda xz$ $\qquad$ $1 = \lambda xy$

$\lambda = 1/yz$ $\qquad$ $1 = \frac{1}{yz} \cdot xz = 1 = x/y$ $\qquad$ $1 = \frac{1}{yz} \cdot xy$

$x = y$ $\qquad$ $1 = x/z \rightarrow x = z$

$g(x,y,z) = xyz = 125$

$x \cdot x \cdot x = 125 \rightarrow x^3 = 125 \rightarrow x = 5$ $\qquad$ $5 \cdot 5 z = 125 \rightarrow z = 5$

$y \cdot y \cdot y = 125 \rightarrow y^3 = 125 \rightarrow y = 5$

Extreme Value at $(5,5,5)$ is $f(5,5,5) = 5+5+5 = 15$ max

2. Use Lagrange Multipliers to find the largest value that xyz can be, given that $x + y + z = 15$

$\nabla f(x,y,z) = \langle yz, xz, xy \rangle$ $\qquad$ $\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$

$\nabla g(x,y,z) = \langle 1,1,1 \rangle$ $\qquad$ $\langle yz, xz, xy \rangle = \lambda \langle 1,1,1 \rangle$

$yz = \lambda$ $\qquad$ $xz = \lambda \rightarrow xz = yz$ $\qquad$ $xy = \lambda = y^2 = yz$

$x = y$ $\qquad$ $y = z$

$y + y + y = 15$ $\qquad$ $x + 5 + x = 15$ $\qquad$ $5 + 5 + z = 15$

$3y = 15 \rightarrow y = 5$ $\qquad$ $2x = 10 \rightarrow x = 5$ $\qquad$ $z = 5$

Extreme value at $(5,5,5)$ is $f(5,5,5) = 5 \cdot 5 \cdot 5 = 125$ max