

$$1. \quad f(x, y, z) = x + y + z \quad xyz = 125$$

$$\nabla f = \langle 1, 1, 1 \rangle$$

$$\nabla g = \langle yz, xz, xy \rangle$$

$$\nabla f = \lambda \nabla g$$

$$1 = \lambda yz, \quad 1 = \lambda xz, \quad 1 = \lambda xy, \quad xyz = 125$$

$$\lambda^3 (xyz)^2 = 1, \quad (xyz)^2 = 15625$$

$$\lambda^3 = \frac{1}{15625}$$

$$\lambda = \frac{1}{25}$$

$$x = y = z = 5 \Rightarrow \text{the point } (5, 5, 5)$$

$$f(5, 5, 5) = 15$$

the smallest value is 15

$$2. \quad f(x, y, z) = xyz, \quad x + y + z = 15$$

$$\nabla f = \langle yz, xz, xy \rangle \quad \nabla g = \langle 1, 1, 1 \rangle$$

$$\nabla f = \lambda \nabla g \quad | \quad \nabla f = \lambda \nabla g$$

$$1 = \lambda yz, \quad 1 = \lambda xz, \quad 1 = \lambda xy, \quad x + y + z = 15$$

$$z = \frac{1}{\lambda y}, \quad \frac{1}{\lambda z} = x, \quad \frac{1}{\lambda x} = y \quad | \quad \begin{array}{l} yz = \lambda, \quad xz = \lambda, \quad xy = \lambda \\ xz = xy = yz = \lambda \end{array}$$

$$\frac{1}{\lambda y} + \frac{1}{\lambda z} + \frac{1}{\lambda x} = 15$$

$$\frac{1}{\lambda} \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = 15$$

$$\lambda = 25$$

$$x = 5, \quad y = 5, \quad z = 5$$

$$f(5, 5, 5) = 25$$

the largest value is 25

