"QUIZ" for Lecture 11

NAME: (print!) Irina Mukhametzhanova **Section:** 24

E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q11FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 12, 8:00pm Deadline extended to Oct. 17

1. Use Largange multipliers (no credit for other methods) to find the **smallest** value that $x+y+z$ can be, given that $xyz = 125$

First, we need to find the gradients of both functions:

$\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle = \langle 1,1,1 \rangle$ $\nabla g = \langle \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z} \rangle = \langle yz, xz, xy \rangle$

We then introduce a constant variable λ, so that ∇f is a multiple of ∇g, like this

$\nabla f = \lambda \nabla g \to \langle 1,1,1 \rangle = \lambda \langle yz, xz, xy \rangle \to \lambda yz = 1, \lambda xz = 1, \lambda xy = 1$

We have 4 equations and 4 unknowns. So, we solve this system of equations:

Multiply first 3 equations: $\lambda^3 (yz \cdot xz \cdot xy) = (xyz)^2 \lambda^3 = 1^3 = 1$

$xyz = 125$, so $\to (125)^2 \cdot \lambda^3 = 1 \to \lambda = \sqrt[3]{\frac{1}{125^2}} \to \lambda = \frac{1}{25}$

$\frac{(xyz)^2 \lambda^3}{z^2} = \frac{1}{z^2} \to (xy)^2 (\frac{1}{25^3}) = \frac{1}{z^2} \Rightarrow (1)^2 \frac{1}{25^3} = \frac{1}{z^2} \to z^2 = \frac{1}{25^3} \to z = \pm \frac{1}{125}$

Because $xy = yz = xy = 1$, $x = \pm \frac{1}{125}$, and $y = \pm \frac{1}{125}$

The minimum value is the sum of all negative values of $x, y,$ and $z \to (-\frac{1}{125}) \cdot 3 = \boxed{-\frac{3}{125}}$

2. Use Largange multipliers (no credit for other methods) to find the **largest** value that xyz can be, given that $x+y+z = 15$

First, we need to find the gradients of both functions:

$\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle = \langle yz, xz, xy \rangle$ $\nabla g = \langle \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z} \rangle = \langle 1,1,1 \rangle$

We then introduce a constant variable λ, so that ∇f is a multiple of ∇g, like this:

$\nabla f = \lambda \nabla g \to \langle yz, xz, xy \rangle = \lambda \langle 1,1,1 \rangle \Rightarrow yz = \lambda, xz = \lambda, xy = \lambda$

We have 4 equations and 4 unknowns. So, we solve this system of equations.

Multiply first 3 equations: $(xyz)^2 = \lambda^3 xyz$, so, $xyz = \lambda^3$

$yz = \frac{xyz}{x} = \frac{\lambda^3}{x} = \lambda \to x = \lambda^2$, $xz = \frac{xyz}{y} = \frac{\lambda^3}{y} = \lambda \to y = \lambda^2$,

$xy = \frac{xyz}{z} = \frac{\lambda^3}{z} = \lambda \to z = \lambda^2$

Plug them into g:

$x+y+z = 3\lambda^2 = 15 \to \lambda^2 = 5 \to \lambda = \pm\sqrt{5}$

Our values for $x, y,$ and z are $\to x = \lambda^2 = 5$, $y = \lambda^2 = 5$, $z = \lambda^2 = 5$

Plug back into $f \to f(5,5,5) = 5 \cdot 5 \cdot 5 = 125 \to$ $\boxed{\text{The maximum value is } 125}$