

Yash Khangura "Quiz" for Lecture 10

Section 24

1. Find the local maximum and minimum point(s), the local maximum and minimum values, and saddle point(s) of the function $f(x,y) = 12x^2 - 4x^3 + 6y^2 + 12xy$

$$f_x(x,y) = 24x - 12x^2 + 12y = 0 \quad f_y(x,y) = 12y + 12x = 0$$

$$y = \frac{12x^3 - 24x}{12} = x(x^2 - 2) \quad y = -x$$

Critical Points: $(0,0)$; $(1,-1)$

$$D = \begin{vmatrix} f_{xx}(x,y) = 24 - 36x^2 & f_{xy}(x,y) = 12 \\ f_{yx}(x,y) = 12 & f_{yy}(x,y) = 12 \end{vmatrix} = f_{xx} \cdot f_{yy} - (f_{xy})^2$$

$(0,0)$: $(24)(12) - (12)^2 = 144 \rightarrow f(0,0)$ is a local minimum

$(1,-1)$: $(-12)(12) - (12)^2 = -288 \rightarrow f(1,-1)$ is a saddle point

$f(0,0) = 0$ is a local minimum, and $f(1,-1) = 2$ is a saddle point