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Section 22

Find local max & min point(s), local max & min values, and saddle point(s) of function

$f(x,y) = 12x^2 - 4x^3 + 6y^2 + 12xy$

$f_x = 24x - 12x^2 + 12y$

$f_y = 12y + 12x$

$f_{xx} = 24 - 24x$

$f_{xy} = 12$

$f_{yy} = 12$

$f_x = 0 \rightarrow 24x - 12x^2 + 12y = 0$

$f_y = 0 \rightarrow 12y + 12x = 0$

$\frac{12y}{-12} = \frac{-12x}{-12}$

$-y = x$

$24(-y) - 12(-y)^2 + 12y = 0$

$-24y - 12y^2 + 12y = 0$

$-12y^2 - 12y = 0$

$-12y(y+1) = 0$

$y = 0, y = 1$

$x = 0, -1$

Critical Points: $(0,0)$, $(0,-1)$, $(1,0)$, $(1,-1)$

$f_{xx}(0,0) = 24$

$f_{xy}(0,0) = 12$

$f_{yy}(0,0) = 12$

$D = (24)(12) - (12)^2 = 144$

Because $f_{xx} = 24 > 0$

$(0,0)$ is a local minimum

$f_{xx}(0,-1) = -12$

$f_{xy}(0,-1) = 12$

$f_{yy}(0,-1) = 12$

$D = (-12)(12) - (12)^2$

$= -288 < 0$

so $(0,-1)$ is a saddle point

$f_{xx}(1,0) = 0$

$f_{xy}(1,0) = 12$

$f_{yy}(1,0) = 12$

$D = 0(12) - (12)^2$

$= -144$

Thus $(1,0)$ is a saddle point

$f_{xx}(1,-1) = 48$

$f_{xy}(1,-1) = 12$

$f_{yy}(1,-1) = 12$

$D = 48(12) - (12)^2 = 432$

Because $f_{xx} > 0$

$(1,-1)$ is a local minimum