"QUIZ" for Lecture 10

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q10FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 8, 8:00pm

1. Find the local maximum and minimum point(s), the local maximum and minimum values, and saddle point(s) of the function

$$f(x,y) = 12x^2 - 4x^3 + 6y^2 + 12xy \quad .$$

$f_x = 24x - 12x^2 + 12y$

$f_y = 12y + 12x$

$f_{xx} = 24 - 24x$

$f_{xy} = 12$

$f_{yy} = 12$

$24x - 12x^2 + 12y = 0$

$12y + 12x = 0$

$24x - 12x^2 + 12y = 0$

$-12x + 0x^2 - 12y = 0$

$12x - 12x^2 = 0$

$x(12 - 12x) = 0$

$x = 0 \quad x = 1$

$12y + 0 = 0 \quad 12y + 12 = 0$

$y = 0 \quad y = -1$

$(0,0) \quad (1,-1)$

$f_{xx}(0,0) = 24$ $f_{xy}(0,0) = 12$ $f_{yy}(0,0) = 12$

$f_{xx}(1,-1) = 0$ $f_{xy}(1,-1) = 12$ $f_{yy}(1,-1) = 12$

$D(0,0) = 24(12) - (12)^2 = 144 \rightarrow$ local min at $(0,0)$

$D(1,-1) = 0(12) - (12)^2 = -144 \rightarrow$ saddle point at $(1,-1)$

$S = \{(x,y) \mid 0 \geq x \geq 1 \, , \, -1 \geq y \geq 0\}$

Left: $f(0,y) = 6y^2$

$f'(y) = 12y = 0$

$y = 0$

$f(0) = 0$

$f(-1) = 6$

$f(0) = 0$

Right: $f(1,y) = 12 - 4 + 6y^2 + 12y$

$= 6y^2 + 12y + 8$

$f'(y) = 12y + 12 = 0$

$y = -1$

$f(-1) = 6 - 12 + 8 = 2$

$f(-1) = 2$

$f(0) = 8$

Down:

$f(x,-1) = 12x^2 - 4x^3 + 6 - 12x$

$= -4x^3 + 12x^2 - 12x + 6$

$f'(x) = -12x^2 + 24x - 12 = 0$

$-12(x^2 - 2x + 1) = 0$

$(x-1)(x-1) = 0$

$x = 1$

$f(1) = -4 + 12 - 12 + 6 = 2$

$f(0) = 6$

$f(1) = 2$

Up:

$f(x,0) = 12x^2 - 4x^3$

$f'(x) = 24x - 12x^2 = 0$

$12x(2-x) = 0$

$12x = 0 \quad 2 - x = 0$

$x = 0 \quad x = 2$

$f(0) = 0$

$f(2) = 48 - 32 = 16$

$f(0) = 0$

$f(1) = 12 - 4 = 8$

Candidates: 6, 0, 2, 8, 6, 2, 16, 0

0 ↓ local minimum value

16 ↓ local maximum value

ANSWERS:

local minimum at (0,0)

saddle point at (1,-1)

local minimum = 0

local maximum = 16