1. $f(x,y) = 12x^2 - 4x^3 + 6y^2 + 12xy$

$f_x(x,y) = 24x - 12x^2 + 12y$

$f_y(x,y) = 12y + 12x$

$f_{xx}(x,y) = 24 - 24x$ ~~$+12y$~~

$f_{xy}(x,y) = 12$

$f_{yy}(x,y) = 12$

$24x - 12x^2 + 12y = 0$, $12y + 12x = 0$

$12(2x - x^2 + y) = 0$, $12(x+y) = 0$

$2x - x^2 - x = 0$, $x = -y$

$x - x^2 = 0$

$x(x-1) = 0$

$x = 0$ or $x = 1 \Rightarrow f_x(0,y) = 0$, $y = 0$

$f_x(1,y) = 24 - 12 + 12y = 0$, $y = -1$

Critical points are $(0,0)$ and $(1,-1)$

when $(x,y) = (0,0)$

$f_{xx}(x,y) = 24$

$f_{xy}(x,y) = 12$

$f_{yy}(x,y) = 12$

$D_1 = 24 \cdot 12 - 12^2 = 144$

$D_1 > 0$ and $f_{xx}(0,0) > 0$

$f(0,0)$ is a local minimum.

when $(x,y) = (1,-1)$

$f_{xx}(x,y) = 0$

$f_{xy}(x,y) = 12$

$f_{xy}(x,y) = 12$

$D_2 = 0 \cdot 12 - 12^2 = -144$

$D_2 < 0$

$f(1,-1)$ is saddle point.