"QUIZ" for Lecture 10

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E-MAIL SCANNED .pdf OF COMPLETED QUIZ to DrZcalc3@gmail.com (Attachment: q10FirstLast.pdf) ASAP BUT NO LATER THAN Oct. 8, 8:00pm

1. Find the local maximum and minimum point(s), the local maximum and minimum values, and saddle point(s) of the function

$$f(x,y) = 12x^2 - 4x^3 + 6y^2 + 12xy \quad .$$

First, we need to find the first-order derivatives:

$f_x = 24x - 12x^2 + 12y$ $\quad f_y = 12y + 12x$

Also, find all the second-order derivatives for future use:

$f_{xx} = 24 - 24x$ $\quad f_{xy} = 12$ $\quad f_{yy} = 12$

We first need to find all of the critical points, or where both first-order derivatives are 0 — there is a possible min, max, or saddle point:

$24x - 12x^2 + 12y = 0 \longrightarrow 24x - 12x^2 - 12x = 0$

$12y + 12x = 0 \rightarrow y = -x \longrightarrow -12x^2 + 12x = 0$

$x^2 - x = 0$

$x(x-1) = 0 \rightarrow x = 0, 1 \rightarrow y = 0, -1$

The critical points are $(0,0)$ and $(1,-1)$

Now, to determine if they are min, max, or a saddle point, we use a determinant:

$D = f_{xx} f_{yy} - f_{xy}^2 \rightarrow$

If $D > 0$ and $f_{xx} > 0$, it is a local max

If $D > 0$ and $f_{xx} < 0$, it is a local min

If $D < 0$, it is a saddle point

If $D = 0$, we do not know

$D(0,0) = (24 - 24(0))(12) - 12^2 = (24)(12) - 144 = 144$

$f_{xx}(0,0) = 24 - 24(0) = 24 \Rightarrow$ (0,0) is a local max

$D(1,-1) = (24 - 24(1))(12) - 12^2 = -144 \Rightarrow$ (1,-1) is a saddle point