L10: 14.7 pdf Quiz

10/8/20

1. Find the local max. & min. pt(s)., the local max. & min. vals., & saddle pt(s). of the fn:

$$f(x,y) = 12x^2 - 4x^3 + 6y^2 + 12xy$$

Step 1: Find crit. pts. (set 1^{st}-order derivs. = 0)

$f_x(x,y) = 24x - 12x^2 + 12y = 0$ (1) sub.

$f_y(x,y) = 12y + 12x = 0$ (2) $x = -y$

$24(-y) - 12(-y)^2 + 12y = 0$

$-24y - 12y^2 + 12y = 0$

$-12y^2 - 12y = 0$

$-12y(y+1) = 0$

$y = 0, -1$

$x = -(0) = 0$

$x = -(-1) = 1$

$x = 0, 1$

Crit. pts.: $(0,0)$, $(1,-1)$

Step 2: Compute discriminant (2^{nd}-order partial derivs.)

$f_{xx}(x,y) = 24 - 24x$

$f_{yy}(x,y) = 12$

$f_{xy}(x,y) = 12$

$D(x,y) = f_{xx}f_{yy} - f_{xy}^2$

$= (24-24x)(12) - (12)^2$

$= 288 - 288x - 144 = -288x + 144$

$= -144(2x-1)$

Step 3: 2^{nd} Deriv. Test

$D(0,0) = 144 > 0 \Rightarrow f_{xx}(0,0) = 24 > 0$ (local min.)

$D(1,-1) = -144 < 0 \Rightarrow$ (saddle pt.)

$(1,-1)$ is a saddle pt. & $f(0,0)$ is a local min. w/ val. of 0