33. $A = \frac{\pi}{16}$ **35.** $e - \frac{1}{e}$
Note: One needs to double the integral from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ in order to account for both sides of the graph.

37. $A = \frac{3\pi a^2}{2}$

39. Outer: $L \approx 36.121$, inner: $L \approx 7.5087$, difference: 28.6123

41. Ellipse. Vertices: $(\pm 3, 0)$, $(0, \pm 2)$. Foci: $(\pm\sqrt{5}, 0)$.

43. Ellipse. Vertices: $\left(\pm\frac{2}{\sqrt{5}}, 0\right)$, $\left(0, \pm\frac{4}{\sqrt{5}}\right)$. Foci: $\left(0, \pm\sqrt{\frac{12}{5}}\right)$.

45. $\left(\frac{x}{8}\right)^2 + \left(\frac{y}{\sqrt{61}}\right)^2 = 1$ **47.** $\left(\frac{x}{8}\right)^2 - \left(\frac{y}{6}\right)^2 = 1$ **49.** $x = \frac{1}{32}y^2$

51. $y = \sqrt{3}x + \left(\sqrt{3} - 5\right)$ and $y = -\sqrt{3}x + \left(-\sqrt{3} - 5\right)$

Chapter 12

Section 12.1 Preliminary Questions

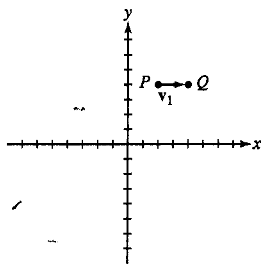
1. **(a)** True **(b)** False **(c)** True **(d)** True

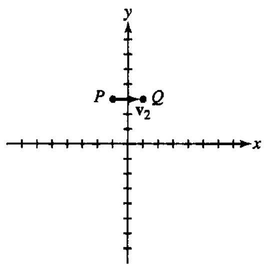
2. $\|-3\mathbf{a}\| = 15$ **3.** The components are not changed. **4.** $\langle 0, 0\rangle$

5. **(a)** True **(b)** False

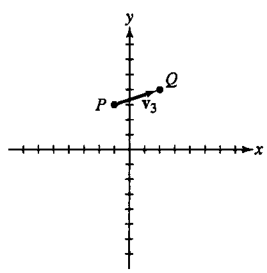
Section 12.1 Exercises

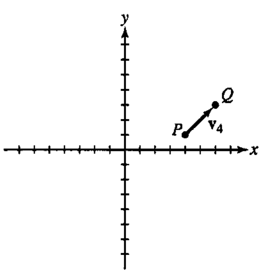
1. $\mathbf{v}_1 = \langle 2, 0\rangle$, $\|\mathbf{v}_1\| = 2$ $\mathbf{v}_2 = \langle 2, 0\rangle$, $\|\mathbf{v}_2\| = 2$





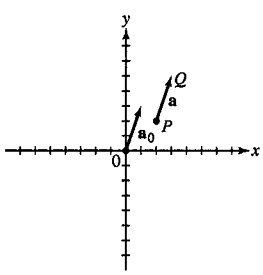
$\mathbf{v}_3 = \langle 3, 1\rangle$, $\|\mathbf{v}_3\| = \sqrt{10}$ $\mathbf{v}_4 = \langle 2, 2\rangle$, $\|\mathbf{v}_4\| = 2\sqrt{2}$





Vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are equivalent.

3. $\langle 3, 5\rangle$



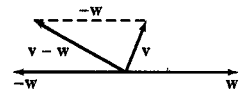
5. $\left\langle \frac{\sqrt{2}}{2}\|\mathbf{u}\|, \frac{\sqrt{2}}{2}\|\mathbf{u}\|\right\rangle$ or $\langle 0.707\|\mathbf{u}\|, 0.707\|\mathbf{u}\|\rangle$

7. $\langle \cos(-20°)\|\mathbf{w}\|, \sin(-20°)\|\mathbf{w}\|\rangle$ or $\langle 0.94\|\mathbf{w}\|, -0.342\|\mathbf{w}\|\rangle$

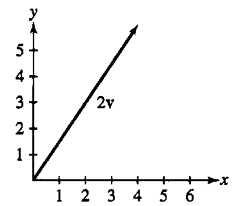
9. $\overrightarrow{PQ} = \langle -1, 5\rangle$ **11.** $\overrightarrow{PQ} = \langle -2, -9\rangle$ **13.** $\langle 5, 5\rangle$

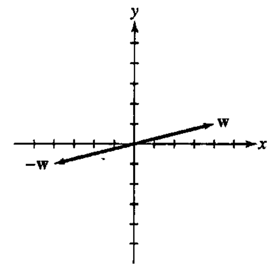
15. $\langle 30, 10\rangle$ **17.** $\left\langle \frac{5}{2}, 5\right\rangle$

19. Vector (B)

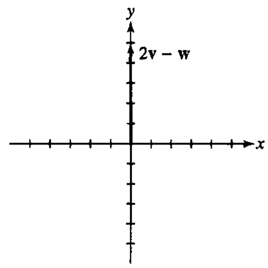


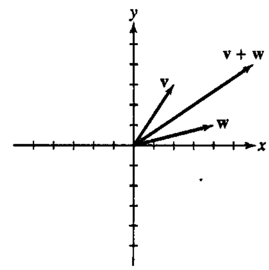
21. $2\mathbf{v} = \langle 4, 6\rangle$ $-\mathbf{w} = \langle -4, -1\rangle$



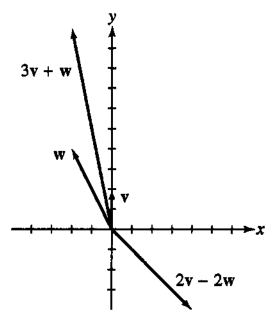


$2\mathbf{v} - \mathbf{w} = \langle 0, 5\rangle$ $\mathbf{v} + \mathbf{w} = \langle 6, 4\rangle$

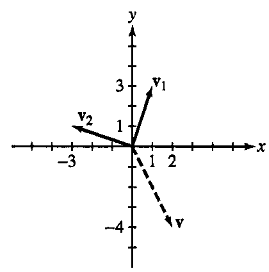




23. $3\mathbf{v} + \mathbf{w} = \langle -2, 10\rangle$, $2\mathbf{v} - 2\mathbf{w} = \langle 4, -4\rangle$



25.



27. (b) and (c)

29. $\overrightarrow{AB} = \langle 2, 6\rangle$ and $\overrightarrow{PQ} = \langle 2, 6\rangle$; equivalent

31. $\overrightarrow{AB} = \langle 3, -2\rangle$ and $\overrightarrow{PQ} = \langle 3, -2\rangle$; equivalent

33. $\overrightarrow{AB} = \langle 2, 3\rangle$ and $\overrightarrow{PQ} = \langle 6, 9\rangle$; parallel and point in the same direction

35. $\overrightarrow{AB} = \langle -8, 1\rangle$ and $\overrightarrow{PQ} = \langle 8, -1\rangle$; parallel and point in opposite directions

37. $\left\|\overrightarrow{OR}\right\| = \sqrt{53}$ **39.** $P = (0, 0)$ **41.** $\left\langle \frac{3}{5}, \frac{4}{5}\right\rangle$

43. $4\mathbf{e}_{\mathbf{u}} = \left\langle -2\sqrt{2}, -2\sqrt{2}\right\rangle$ **45.** $2\mathbf{e}_{-\mathbf{v}} = -\sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j}$

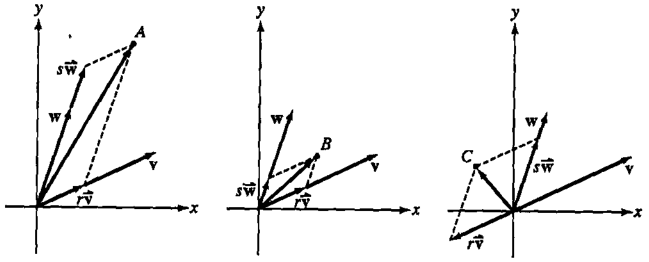
47. $\mathbf{e} = \left\langle \cos\frac{4\pi}{7},\ \sin\frac{4\pi}{7} \right\rangle = \langle -0.22,\ 0.97 \rangle$

49. $\lambda = \pm\frac{1}{\sqrt{13}}$ **51.** $P = (4,\ 6)$

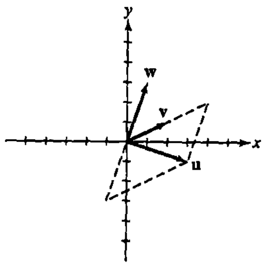
53. (a) → (ii), (b) → (iv), (c) → (iii), (d) → (i) **55.** $9\mathbf{i} + 7\mathbf{j}$

57. $-5\mathbf{i} - 3\mathbf{j}$

59.



61. $\mathbf{u} = 2\mathbf{v} - \mathbf{w}$



63. The force on cable 1 is ≈ 45 lb, and force on cable 2 is ≈ 21 lb.

65. 230 km/h **67.** $\mathbf{r} = \langle 6.45,\ 0.38 \rangle$

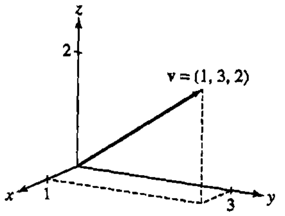
Section 12.2 Preliminary Questions

1. $\langle 4,\ 3,\ 2 \rangle$ **2.** $\langle 3,\ 2,\ 1 \rangle$ **3.** (a) **4.** (c)

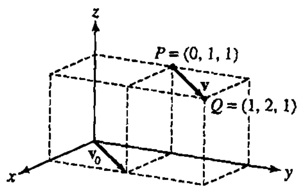
5. Infinitely many direction vectors **6.** True

Section 12.2 Exercises

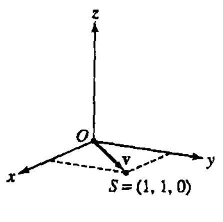
1. $\|\mathbf{v}\| = \sqrt{14}$



3. The head of $\mathbf{v} = \overrightarrow{PQ}$ is $Q = (1,\ 2,\ 1)$.



$\mathbf{v}_0 = \overrightarrow{OS}$, where $S = (1,\ 1,\ 0)$



5. $\overrightarrow{PQ} = \langle 1,\ 1,\ -1 \rangle$ **7.** $\overrightarrow{PQ} = \left\langle -\frac{9}{2},\ -\frac{3}{2},\ 1 \right\rangle$

9. $\left\|\overrightarrow{OR}\right\| = \sqrt{26} \approx 5.1$ **11.** $P = (-2,\ 6,\ 0)$

13. **(a)** Parallel and same direction **(b)** Not parallel
(c) Parallel and opposite directions **(d)** Not parallel

15. Not equivalent **17.** Not equivalent **19.** $\langle -8,\ -18,\ -2 \rangle$

21. $\langle -2,\ -2,\ 3 \rangle$ **23.** $\langle 16,\ -1,\ 9 \rangle$

25. Not parallel **27.** Not parallel

29. $\mathbf{e}_{\mathbf{w}} = \left\langle \frac{4}{\sqrt{21}},\ \frac{-2}{\sqrt{21}},\ \frac{-1}{\sqrt{21}} \right\rangle$ **31.** $-\mathbf{e}_{\mathbf{v}} = \left\langle \frac{2}{3},\ -\frac{2}{3},\ -\frac{1}{3} \right\rangle$

33. $\mathbf{r}(t) = \langle 1+2t,\ 2+t,\ -8+3t \rangle$

35. $\mathbf{r}(t) = \langle 4+7t,\ 0,\ 8+4t \rangle$ **37.** $\mathbf{r}(t) = \langle 1+2t,\ 1-6t,\ 1+t \rangle$

39. $\mathbf{r}(t) = \langle 4t,\ t,\ t \rangle$ **41.** $\mathbf{r}(t) = \langle 0,\ 0,\ t \rangle$

43. $\mathbf{r}(t) = \langle -t,\ -2t,\ 4-2t \rangle$ **45.** (c)

49. $\mathbf{r}_1(t) = \langle 5,\ 5,\ 2 \rangle + t\,\langle 0,\ -2,\ 1 \rangle$;
$\mathbf{r}_2(t) = \langle 5,\ 5,\ 2 \rangle + t\,\langle 0,\ -20,\ 10 \rangle$

55. 4 min **57.** $\left\langle 0, \frac{1}{2}, -\frac{1}{2} \right\rangle$ **59.** 2450 newtons

61. $\frac{x+2}{2} = \frac{y-3}{4} = \frac{z-3}{3}$ **63.** $\frac{x-3}{2} = \frac{y-4}{-9} = \frac{z}{12}$

65. $\mathbf{r}(t) = \langle 2t, 7t, 8t \rangle$

Section 12.3 Preliminary Questions

1. Scalar **2.** Obtuse **3.** Distributive Law

4. (a) **5.** (b); (c) **6.** (c)

Section 12.3 Exercises

1. 15 **3.** 41 **5.** 5 **7.** 0 **9.** 1 **11.** 0 **13.** Obtuse

15. Orthogonal **17.** Acute **19.** 0 **21.** $\frac{1}{\sqrt{10}}$ **23.** $\pi/4$

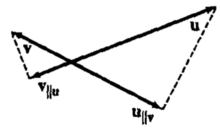
25. ≈0.615 **27.** $2\pi/3$

29. (a) $b = -\frac{1}{2}$ (b) $b = 0$ or $b = \frac{1}{2}$

31. $\mathbf{v}_1 = \langle 0,\ 1,\ 0 \rangle$, $\mathbf{v}_2 = \langle 3,\ 2,\ 2 \rangle$ **33.** $-\frac{3}{2}$ **35.** $\|\mathbf{v}\|^2$

37. $\|\mathbf{v}\|^2 - \|\mathbf{w}\|^2$ **39.** 8 **41.** 2 **43.** π **45.** (b) 7 **49.** 51.91°

51. (a)



(b) $\mathbf{u}_{\|\mathbf{v}}$

53. $\left\langle \frac{7}{2},\ \frac{7}{2} \right\rangle$ **55.** $\left\langle -\frac{4}{5},\ 0,\ -\frac{2}{5} \right\rangle$ **57.** $-4\mathbf{k}$ **59.** $a\mathbf{i}$ **61.** $2\sqrt{2}$

63. $\sqrt{17}$ **65.** $\mathbf{a} = \left\langle \frac{1}{2},\ \frac{1}{2} \right\rangle + \left\langle \frac{1}{2},\ -\frac{1}{2} \right\rangle$

67. $\mathbf{a} = \left\langle 0,\ -\frac{1}{2},\ -\frac{1}{2} \right\rangle + \left\langle 4,\ -\frac{1}{2},\ \frac{1}{2} \right\rangle$

69. $\left\langle \frac{x-y}{2},\ \frac{y-x}{2} \right\rangle + \left\langle \frac{x+y}{2},\ \frac{y+x}{2} \right\rangle$

73. ≈35° **75.** $\overrightarrow{AD}$ **77.** ≈109.5° **81.** ≈68.07 N

99. $2x + 2y - 2z = 1$

Section 12.4 Preliminary Questions

1. $\begin{vmatrix} -5 & -1 \\ 4 & 0 \end{vmatrix}$ **2.** $\|\mathbf{e} \times \mathbf{f}\| = \frac{1}{2}$ **3.** $\mathbf{u} \times \mathbf{v} = \langle -2,\ -2,\ -1 \rangle$

4. (vector notation) **(a)** 0 **(b)** 0

5. $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ and $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$

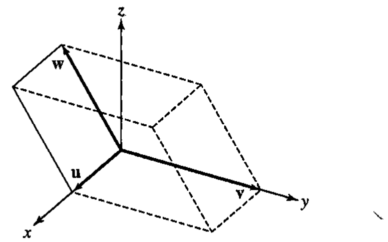
6. $\mathbf{v} \times \mathbf{w} = \mathbf{0}$ if either $\mathbf{v}$ or $\mathbf{w}$ (or both) is the zero vector or $\mathbf{v}$ and $\mathbf{w}$ are parallel vectors.

7. **(a)** not meaningful because you cannot find the cross product of a scalar and a vector;
(b) meaningful because it represents the dot product of two vectors;
(c) meaningful because it is the product of two scalars;
(d) meaningful because it is a scalar multiple of a vector.

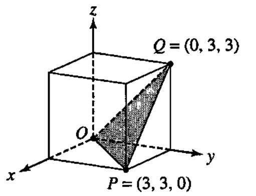
8. (b)

ction 12.4 Exercises

. -5 **3.** -15 **5.** -8 **7.** 0 **9.** $\langle 1, 2, -5\rangle$ **11.** $\langle 6, 0, -8\rangle$
3. $-\mathbf{j}+\mathbf{i}$ **15.** $\mathbf{i}+\mathbf{j}+\mathbf{k}$ **17.** $\langle -1, -1, 0\rangle$
19. $\langle -2, -2, -2\rangle$ **21.** $\langle 4, 4, 0\rangle$
23. $\mathbf{v}\times\mathbf{i} = c\mathbf{j} - b\mathbf{k}$; $\mathbf{v}\times\mathbf{j} = -c\mathbf{i} + a\mathbf{k}$; $\mathbf{v}\times\mathbf{k} = b\mathbf{i} - a\mathbf{j}$
25. $-\mathbf{u}$ **27.** $\langle 0, 3, 3\rangle$ **31.** $\mathbf{e}'$ **33.** $\mathbf{F}_1$ **37.** $2\sqrt{138}$
39. The volume is 4.



41. $\sqrt{35} \approx 5.92$
43.



The area of the triangle is $\frac{9\sqrt{3}}{2} \approx 7.8$.
45. 3 **47.** $\frac{33}{2}\sqrt{3}$ **59.** $\mathbf{X} = \langle a, a, a+1\rangle$
63. $\tau = 250 \sin 125^\circ\, \mathbf{k} \approx 204.79\,\mathbf{k}$

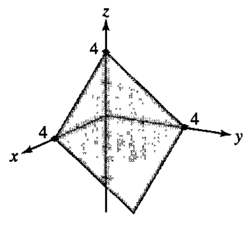
Section 12.5 Preliminary Questions

1. $3x + 4y - z = 0$ **2.** (c): $z = 1$ **3.** Plane (c) **4.** xz-plane
5. (c): $x + y = 0$ **6.** Statement (a)

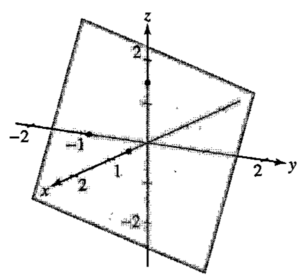
Section 12.5 Exercises

1. $x + 3y + 2z = 3$ **3.** $-x + 2y + z = 3$ **5.** $x = 3$
7. $z = 2$ **9.** $x = 0$ **11.** Statements (b) and (d)
13. $\langle 9, -4, -11\rangle$ **15.** $\langle 3, -8, 11\rangle$ **17.** $6x + 9y + 4z = 19$
19. $x + 2y - z = 1$ **21.** $4x - 9y + z = 0$ **23.** $x = 4$
25. $x + z = 3$ **27.** $13x + y - 5z = 27$ **29.** Yes, the planes are parallel.

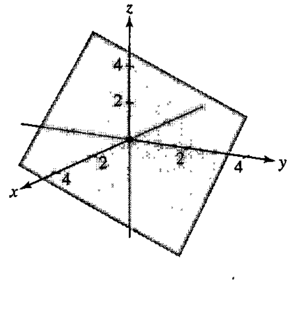
31.



33.



35.



37. $10x + 15y + 6z = 30$ **39.** $(1, 5, 8)$ **41.** $(-2, 3, 12)$
43. $-9y + 4z = 5$ **45.** $x = -\frac{2}{3}$ **47.** $x = -4$
49. The two planes have no common points.
51. $y - 4z = 0$
$x + y - 4z = 0$
53. $(3\lambda)x + by + (2\lambda)z = 5\lambda,\ \lambda \neq 0$ **55.** $\theta = \pi/2$
57. $\theta = 1.143$ rad or $\theta = 65.49^\circ$ **59.** $\theta \approx 55.0^\circ$
61. $x + y + z = 1$ **63.** $x - y - z = d/a$
65. $x = \frac{9}{5} + 2t,\ y = -\frac{6}{5} - 3t,\ z = 2 + 5t$ **67.** $\pm 24\langle 1, 2, -2\rangle$
73. $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$ **75.** $\frac{6}{\sqrt{30}} \approx 1.095$ **77.** $|a|$

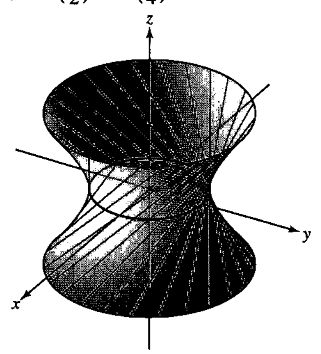
Section 12.6 Preliminary Questions

1. True, mostly, except at $x = \pm a$, $y = \pm b$, or $z = \pm c$
2. False **3.** Hyperbolic paraboloid
4. No **5.** Ellipsoid
6. All vertical lines passing through a parabola c in the xy-plane

Section 12.6 Exercises

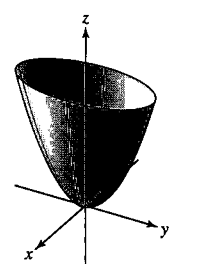
1. Ellipsoid **3.** Ellipsoid
5. Hyperboloid of one sheet **7.** Elliptic paraboloid
9. Hyperbolic paraboloid **11.** Hyperbolic paraboloid
13. Ellipsoid, the trace is a circle on the xz-plane
15. Ellipsoid, the trace is an ellipse parallel to the xy-plane
17. Hyperboloid of one sheet, the trace is a hyperbola
19. Parabolic cylinder, the trace is the parabola $y = 3x^2$
21. (a) ↔ Figure b; (b) ↔ Figure c; (c) ↔ Figure a
23. $y = \left(\frac{x}{2}\right)^2 + \left(\frac{z}{4}\right)^2$

25.

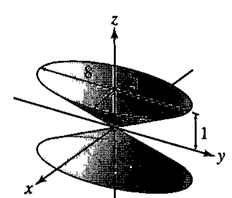


Graph of $x^2 + y^2 + z^2 = 1$

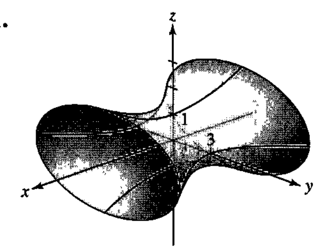
27.



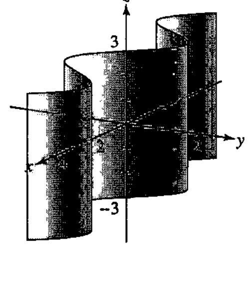
29.



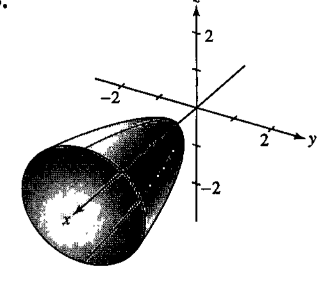
31.



33.



35.



39. $\left(\frac{x}{2}\right)^2+\left(\frac{y}{4}\right)^2+\left(\frac{z}{6}\right)^2=1$ **41.** $\left(\frac{x}{4}\right)^2+\left(\frac{y}{6}\right)^2-\left(\frac{z}{3\sqrt{3}}\right)^2=1$

43. One or two vertical lines, or an empty set **45.** An elliptic cone

Section 12.7 Preliminary Questions

1. Cylinder of radius R whose axis is the z-axis, sphere of radius R centered at the origin

2. (b) **3.** (a) **4.** $\phi=0,\ \pi$ **5.** $\phi=\frac{\pi}{2}$, the xy-plane

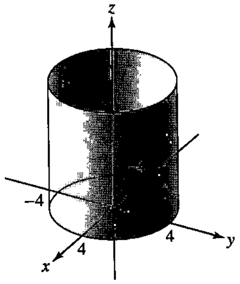
Section 12.7 Exercises

1. $(-4,\ 0,\ 4)$ **3.** $\left(0,\ 0,\ \frac{1}{2}\right)$ **5.** $\left(\sqrt{2},\ \frac{7\pi}{4},\ 1\right)$ **7.** $\left(2,\ \frac{\pi}{3},\ 7\right)$

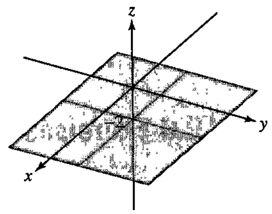
9. $\left(5,\ \frac{\pi}{4},\ 2\right)$ **11.** $r^2\le 1$ **13.** $r^2+z^2\le 4$, $\theta=\frac{\pi}{2}$ or $\theta=\frac{3\pi}{2}$

15. $r^2\le 9$, $\frac{5\pi}{4}\le\theta\le 2\pi$ and $0\le\theta\le\frac{\pi}{4}$

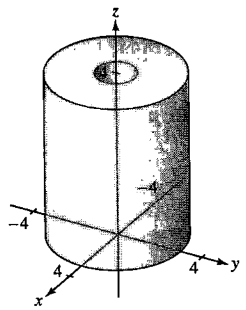
17.



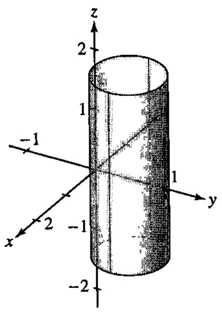
19.



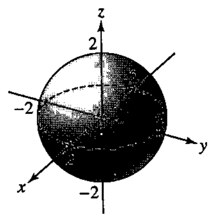
21.



23.



25.



27. $r=\frac{z}{\cos\theta+\sin\theta}$ **29.** $r=\frac{z\tan\theta}{\cos\theta}$ **31.** $r=2$ **33.** $(3,\ 0,\ 0)$

35. $(0,\ 0,\ 3)$ **37.** $\left(\frac{3\sqrt{3}}{2},\ \frac{3}{2},\ -3\sqrt{3}\right)$ **39.** $\left(2,\ 0,\ \frac{\pi}{3}\right)$

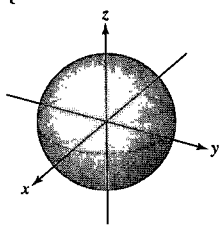
41. $\left(\sqrt{3},\ \frac{\pi}{4},\ 0.955\right)$ **43.** $\left(2,\ \frac{\pi}{3},\ \frac{\pi}{6}\right)$ **45.** $\left(2\sqrt{2},\ 0,\ \frac{\pi}{4}\right)$

47. $\left(2\sqrt{2},\ 0,\ 2\sqrt{2}\right)$ **49.** $0\le\rho\le 1$

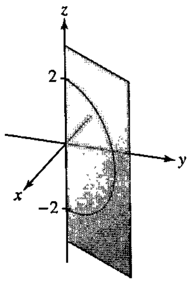
51. $\rho=1$, $0\le\theta\le\frac{\pi}{2}$, $0\le\phi\le\frac{\pi}{2}$

53. $\left\{(\rho,\ \theta,\ \phi): 0\le\rho\le 2,\ \theta=\frac{\pi}{2}\text{ or }\theta=\frac{3\pi}{2}\right\}$

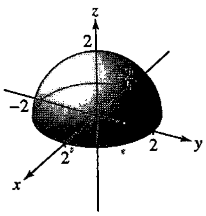
55.



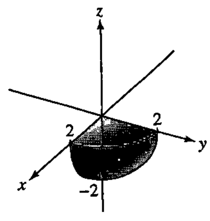
57.



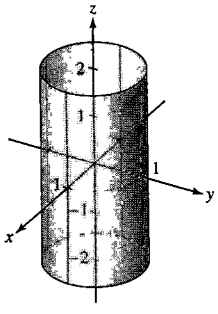
59.



61.



63.



65. $\rho=\frac{2}{\cos\phi}$ **67.** $\rho=\frac{\cos\theta\tan\phi}{\cos\phi}$ **69.** $\rho=\frac{2}{\sin\phi\sqrt{\cos 2\theta}}$ **71.** (b)

73. Helsinki: (25.0°, 29.9°), São Paulo: (313.48°, 113.52°)

75. Sydney: $(-4618.8,\ 2560.3,\ -3562.1)$,
Bogotá: $(1723.7,\ -6111.7,\ 503.1)$

77. $z=\pm r\sqrt{\cos 2\theta}$

79. $\left\{(r,\theta,z): -\sqrt{4-r^2}\le z\le\sqrt{4-r^2}, 1\le r\le 2, 0\le\theta\le 2\pi\right\}$

83. $r=\sqrt{z^2+1}$ and $\rho=\sqrt{-\frac{1}{\cos 2\phi}}$; no points; $\frac{\pi}{4}<\phi<\frac{3\pi}{4}$

Chapter 12 Review

1. $\langle 21,\ -25\rangle$ and $\langle -19,\ 31\rangle$ **3.** $\left\langle\frac{-2}{\sqrt{29}},\ \frac{5}{\sqrt{29}}\right\rangle$

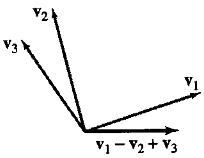
5. $\mathbf{i}=\frac{2}{11}\mathbf{v}+\frac{5}{11}\mathbf{w}$ **7.** $\overrightarrow{PQ}=\langle -4,\ 1\rangle$; $\left\|\overrightarrow{PQ}\right\|=\sqrt{17}$

9. $\left\langle\frac{3}{\sqrt{2}},\ -\frac{3}{\sqrt{2}}\right\rangle$ **11.** $\beta=\frac{3}{2}$ **13.** $\mathbf{u}=\left\langle\frac{1}{3},\ -\frac{11}{6},\ \frac{7}{6}\right\rangle$

15. $\mathbf{r}_1(t)=\langle 1+3t,\ 4+t,\ 5+6t\rangle$; $\mathbf{r}_2(t)=\langle 1+3t,\ t,\ 6t\rangle$

17. $a=-2,\ b=2$

19.



21. $\mathbf{v}\cdot\mathbf{w}=-9$ **23.** $\mathbf{v}\times\mathbf{w}=\langle 10,\ -8,\ -7\rangle$ **25.** $V=48$

29. $\frac{5}{3}$ **31.** $\|\mathbf{F}_1\|=\frac{2\|\mathbf{F}_2\|}{\sqrt{3}}$; $\|\mathbf{F}_1\|=980$ N

33. $\mathbf{v}\times\mathbf{w}=\langle -6,\ 7,\ -2\rangle$

35. -47 **37.** $5\sqrt{2}$ **41.** $\|\mathbf{e}-4\mathbf{f}\|=\sqrt{13}$

47. $(x-0)+4(y-1)-3(z+1)=0$

49. $17x-21y-13z=-28$ **51.** $3x-2y=4$ **53.** Ellipsoid

55. Elliptic paraboloid **57.** Elliptic cone

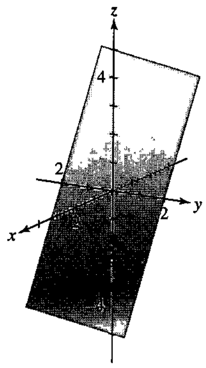
59. **(a)** Empty set **(b)** Hyperboloid of one sheet

(c) Hyperboloid of two sheets

61. $(r,\ \theta,\ z)=\left(5,\ \tan^{-1}\frac{4}{3},\ -1\right)$, $(\rho,\ \theta,\ \phi)=$
$\left(\sqrt{26},\ \tan^{-1}\frac{4}{3},\ \cos^{-1}\left(\frac{-1}{\sqrt{26}}\right)\right)$

63. $(r,\ \theta,\ z)=\left(\frac{3\sqrt{3}}{2},\ \frac{\pi}{6},\ \frac{3}{2}\right)$

$z = 2x$



69. $A < -1$: Hyperboloid of one sheet
$A = -1$: Cylinder with the z-axis as its central axis
$A > -1$: Ellipsoid
$A = 0$: Sphere

Chapter 13

Section 13.1 Preliminary Questions

1. (c) **2.** The curve $z = e^x$

3. The projection onto the xz-plane **4.** The point $(-2, 2, 3)$

5. As t increases from 0 to 2π, a point on $\sin t\mathbf{i} + \cos t\mathbf{j}$ moves clockwise and a point on $\cos t\mathbf{i} + \sin t\mathbf{j}$ moves counterclockwise.

6. (a), (c), and (d)

Section 13.1 Exercises

1. $D = \{t \in \mathbf{R},\ t \neq 0,\ t \neq -1\}$

3. $\mathbf{r}(2) = \left\langle 0,\ 4,\ \frac{1}{5}\right\rangle$; $\mathbf{r}(-1) = \left\langle -1,\ 1,\ \frac{1}{2}\right\rangle$

5. $\mathbf{r}(t) = (3+3t)\mathbf{i} - 5\mathbf{j} + (7+t)\mathbf{k}$

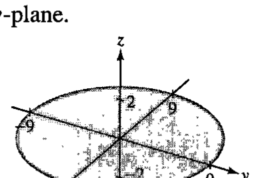
7. Yes, when $t = (2n-1)\pi$, where n is an integer, at the points $(0, 0, (2n-1)\pi)$

9. No intersection **11.** A $\leftrightarrow$ ii, B $\leftrightarrow$ i, C $\leftrightarrow$ iii

13. (a) = (v), (b) = (i), (c) = (ii), (d) = (vi), (e) = (iv), (f) = (iii)

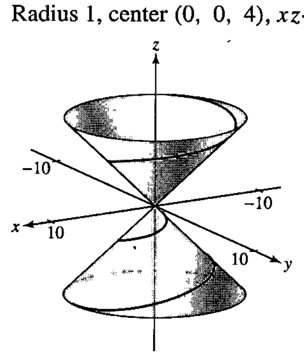
15. C $\leftrightarrow$ i, A $\leftrightarrow$ ii, B $\leftrightarrow$ iii

17. This is a circle of radius 9 centered at the origin lying in the xy-plane.



19. Radius 1, center $(0,\ 0,\ 4)$, xz-plane

21.



23. $(0,\ 1,\ 0), (0,\ -1,\ 0), \left(\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ 0\right), \left(\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}},\ 0\right),$
$\left(-\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}},\ 0\right), \left(-\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ 0\right)$

25. $\mathbf{r}(t) = \left\langle 2t^2 - 7,\ t,\ \pm\sqrt{9-t^2}\right\rangle$, for $-3 \le t \le 3$

27. **(a)** $\mathbf{r}(t) = \left\langle \pm t\sqrt{1-t^2},\ t^2,\ t\right\rangle$ for $-1 \le t \le 1$

(b) The projection is a circle in the xy-plane with radius $\frac{1}{2}$ and centered at the xy-point $\left(0,\ \frac{1}{2}\right)$.

29. $\mathbf{r}(t) = \langle \cos t,\ \pm\sin t,\ \sin t\rangle$; the projection of the curve onto the xy-plane is traced by $\langle \cos t,\ \pm\sin t,\ 0\rangle$, which is the unit circle in this plane; the projection of the curve onto the xz-plane is traced by $\langle \cos t,\ 0,\ \sin t\rangle$, which is the unit circle in this plane; the projection of the curve onto the yz-plane is traced by $\langle 0,\ \pm\sin t,\ \sin t\rangle$, which is the two segments $z = y$ and $z = -y$ for $-1 \le y \le 1$.

31. $\mathbf{r}(t) = \left\langle \cos t,\ \sin t,\ 4\cos t^2\right\rangle$, $0 \le t \le 2\pi$

33. Collide at the point $(12,\ 4,\ 2)$ and intersect at the points $(4,\ 0,\ -6)$ and $(12,\ 4,\ 2)$

35. $\mathbf{r}(t) = \langle 3,\ 2,\ t\rangle$, $-\infty < t < \infty$

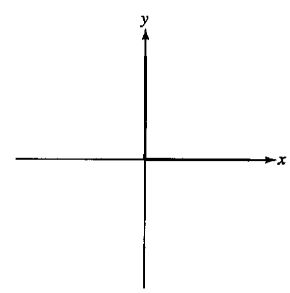
37. $\mathbf{r}(t) = \langle t,\ 3t,\ 15t\rangle$, $-\infty < t < \infty$

39. $\mathbf{r}(t) = \langle 1,\ 2 + 2\cos t,\ 5 + 2\sin t\rangle$, $0 \le t \le 2\pi$

41. $\mathbf{r}(t) = \left\langle \frac{\sqrt{3}}{2}\cos t,\ \frac{1}{2},\ \frac{\sqrt{3}}{2}\sin t\right\rangle$, $0 \le t \le 2\pi$

43. $\mathbf{r}(t) = \langle 3 + 2\cos t,\ 1,\ 5 + 3\sin t\rangle$, $0 \le t \le 2\pi$

45.



$\mathbf{r}(t) = \langle |t| + t,\ |t| - t\rangle$

Section 13.2 Preliminary Questions

1. $\frac{d}{dt}\left(f(t)\mathbf{r}(t)\right) = f(t)\mathbf{r}'(t) + f'(t)\mathbf{r}(t)$

$\frac{d}{dt}\left(\mathbf{r}_1(t) \cdot \mathbf{r}_2(t)\right) = \mathbf{r}_1(t) \cdot \mathbf{r}'_2(t) + \mathbf{r}'_1(t) \cdot \mathbf{r}_2(t)$

$\frac{d}{dt}\left(\mathbf{r}_1(t) \times \mathbf{r}_2(t)\right) = \mathbf{r}_1(t) \times \mathbf{r}'_2(t) + \mathbf{r}'_1(t) \times \mathbf{r}_2(t)$

2. True **3.** False **4.** True **5.** False **6.** False

7. **(a)** Vector **(b)** Scalar **(c)** Vector

Section 13.2 Exercises

1. $\lim_{t\to 3}\left\langle t^2,\ 4t,\ \frac{1}{t}\right\rangle = \left\langle 9,\ 12,\ \frac{1}{3}\right\rangle$

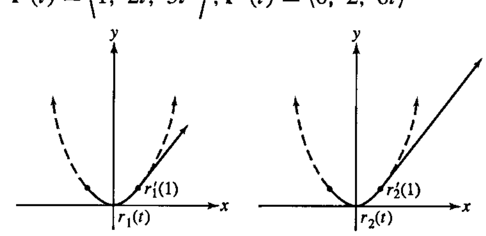
3. $\lim_{t\to 0}\left(e^{2t}\mathbf{i} + \ln(t+1)\mathbf{j} + 4\mathbf{k}\right) = \mathbf{i} + 4\mathbf{k}$

5. $\lim_{h\to 0}\frac{\mathbf{r}(t+h)-\mathbf{r}(t)}{h} = \left\langle -\frac{1}{t^2},\ \cos t,\ 0\right\rangle$ **7.** $\frac{d\mathbf{r}}{dt} = \left\langle 1,\ 2t,\ 3t^2\right\rangle$

9. $\frac{d\mathbf{r}}{ds} = \left\langle 3e^{3s},\ -e^{-s},\ 4s^3\right\rangle$ **11.** $\mathbf{c}'(t) = -t^{-2}\mathbf{i} - 2e^{2t}\mathbf{k}$

13. $\mathbf{r}'(t) = \left\langle 1,\ 2t,\ 3t^2\right\rangle$, $\mathbf{r}''(t) = \langle 0,\ 2,\ 6t\rangle$

15.



17. $\frac{d}{dt}\left(\mathbf{r}_1(t)\cdot\mathbf{r}_2(t)\right) =$
$2t^3e^{2t} + 3t^2e^{3t} + 2te^{3t} + 3t^2e^{2t} + te^t + e^t$

19. $\frac{d}{dt}\left(\mathbf{r}_1(t)\times\mathbf{r}_2(t)\right) =$
$\left\langle \begin{matrix} 3t^2e^t - 2te^{2t} - e^{2t} + t^3e^t, \; e^{3t} + 3te^{3t} - t^2e^t - 2te^t, \\ 2te^{2t} + 2t^2e^{2t} - 3t^2e^{3t} - 3t^3e^{3t} \end{matrix} \right\rangle$

21. $2 + 4e$ **23.** $\frac{d}{dt}\mathbf{r}(g(t)) = \left\langle 2e^{2t}, \; -e^t \right\rangle$

25. $\frac{d}{dt}\mathbf{r}(g(t)) = \left\langle 4e^{4t+9}, \; 8e^{8t+18}, \; 0 \right\rangle$

27. $\frac{d}{dt}\left(\mathbf{r}(t)\cdot\mathbf{a}(t)\right)\big|_{t=2} = 13$ **29.** $\ell(t) = \langle 4 - 4t, \; 16 - 32t \rangle$

31. $\ell(t) = \langle -3 - 4t, \; 10 + 5t, \; 16 + 24t \rangle$

33. $\ell(t) = \left\langle 2 - t, \; 0, \; -\frac{1}{3} + \frac{1}{2}t \right\rangle$

35. $\frac{d}{dt}\left(\mathbf{r}\times\mathbf{r}'\right) = \left\langle (t^2 - 2)e^t, \; -te^t, \; 2t \right\rangle$ **39.** $\left\langle \frac{212}{3}, \; 124 \right\rangle$

41. $\langle 0, \; 0 \rangle$ **43.** $\left\langle 1, \; 2, \; -\frac{\sin 3}{3} \right\rangle$ **45.** $(\ln 4)\mathbf{i} + \frac{56}{3}\mathbf{j} - \frac{496}{5}\mathbf{k}$

47. $\mathbf{r}(t) = \left\langle -t^2 + t + c_1, 2t^2 + c_2 \right\rangle$; with initial conditions
$\mathbf{r}(t) = \left\langle -t^2 + t + 3, 2t^2 + 1 \right\rangle$

49. $\mathbf{r}(t) = \left(\frac{1}{3}t^3\right)\mathbf{i} + \left(\frac{5t^2}{2}\right)\mathbf{j} + t\mathbf{k} + \mathbf{c}$; with initial conditions,
$\mathbf{r}(t) = \left(\frac{1}{3}t^3 - \frac{1}{3}\right)\mathbf{i} + \left(\frac{5}{2}t^2 - \frac{3}{2}\right)\mathbf{j} + (t + 1)\mathbf{k}$

51. $\mathbf{r}(t) = (8t^2)\mathbf{k} + \mathbf{c}_1 t + \mathbf{c}_2$; with initial conditions,
$\mathbf{r}(t) = \mathbf{i} + t\mathbf{j} + (8t^2)\mathbf{k}$

53. $\mathbf{r}(t) = \left\langle 0, \; t^2, \; 0 \right\rangle + \mathbf{c}_1 t + \mathbf{c}_2$; with initial conditions,
$\mathbf{r}(t) = \left\langle 1, \; t^2 - 6t + 10, \; t - 3 \right\rangle$

55. $\mathbf{r}(3) = \left\langle \frac{45}{4}, \; 5 \right\rangle$

57. Only at time $t = 3$ can the pilot hit a target located at the origin.

59. $\mathbf{r}(t) = (t - 1)\mathbf{v} + \mathbf{w}$ **61.** $\mathbf{r}(t) = e^{2t}\mathbf{c}$

Section 13.3 Preliminary Questions

1. $2\mathbf{r}' = \langle 50, \; -70, \; 20 \rangle$, $-\mathbf{r}' = \langle -25, \; 35, \; -10 \rangle$

2. Statement (b) is true.

3. **(a)** $L'(2) = 4$
(b) $L(t)$ is the distance along the path traveled, which is usually different from the distance from the origin.

4. 6

Section 13.3 Exercises

1. $L = 3\sqrt{61}$ **3.** $L = 15 + \ln 4$ **5.** $L = \frac{544\sqrt{34} - 2}{135} \approx 23.48$

7. $L = \pi\sqrt{4\pi^2 + 10} + 5\ln\frac{2\pi + \sqrt{4\pi^2 + 10}}{\sqrt{10}} \approx 29.3$

9. $s(t) = \frac{1}{27}\left((20 + 9t^2)^{3/2} - 20^{3/2}\right)$ **11.** $v(4) = \sqrt{21}$

13. $v(1) = \sqrt{2}$ **15.** $v\left(\frac{\pi}{2}\right) = 5$ **17.** $\mathbf{r}' = \langle 100\sqrt{5}, 200\sqrt{5} \rangle$

19. The bee is at the origin. $\int_0^T \|\mathbf{r}'(u)\|\, du$ represents the total distance the bee traveled on the time interval $[0, T]$.

21. **(c)** $L_1 \approx 132.0, L_2 \approx 125.7$; the first spring uses more wire.

23. **(a)** $t = \pi$

25. **(a)** $s(t) = \sqrt{29}t$ **(b)** $t = g(s) = \frac{s}{\sqrt{29}}$

27. $\left\langle 1 + \frac{3s}{\sqrt{50}}, \; 2 + \frac{4s}{\sqrt{50}}, \; 3 + \frac{5s}{\sqrt{50}} \right\rangle$

29. $\mathbf{r}(s) = \langle 2 + 4\cos(2s), \; 10, \; -3 + 4\sin(2s) \rangle$

31. $\mathbf{r}(s) =$
$\left\langle \cos\left[\left(\frac{3}{2}s + 1\right)^{2/3} - 1\right], \sin\left[\left(\frac{3}{2}s + 1\right)^{2/3} - 1\right], \frac{2}{3}\left[\left(\frac{3}{2}s + 1\right)^{2/3} - 1\right]^{3/2} \right\rangle,$
$s \geq 0.$

33. $\mathbf{r}(s) = \left\langle \frac{1}{9}(27s + 8)^{2/3} - \frac{4}{9}, \; \pm\frac{1}{27}\left((27s + 8)^{2/3} - 4\right)^{3/2} \right\rangle$

35. $\left\langle \frac{s}{\sqrt{1+m^2}}, \; \frac{sm}{\sqrt{1+m^2}} \right\rangle$

37. **(a)** $\sqrt{17}e^t$ **(b)** $\frac{s}{\sqrt{17}}\left\langle \cos\left(4\ln\frac{s}{\sqrt{17}}\right), \; \sin\left(4\ln\frac{s}{\sqrt{17}}\right) \right\rangle$

39. $L = \int_{-\infty}^{\infty} \|\mathbf{r}'(t)\| = 2\int_{-\infty}^{\infty} \frac{dt}{1+t^2} = 2\pi$

Section 13.4 Preliminary Questions

1. $\left\langle \frac{2}{3}, \; \frac{1}{3}, \; -\frac{2}{3} \right\rangle$ **2.** $\frac{1}{4}$

3. The curvature of a circle of radius 2 **4.** Zero curvature

5. $\kappa = \sqrt{14}$ **6.** 4 **7.** $\frac{1}{9}$

Section 13.4 Exercises

1. $\mathbf{r}'(t) = \langle 8t + 9 \rangle$, $\mathbf{T}(t) = \frac{1}{\sqrt{64t^2+81}}\langle 8t, \; 9 \rangle$,
$\mathbf{T}(1) = \left\langle \frac{8}{\sqrt{145}}, \; \frac{9}{\sqrt{145}} \right\rangle$

3. $\mathbf{r}'(t) = \langle 4, -5, 9 \rangle$, $\mathbf{T}(t) = \left\langle \frac{4}{\sqrt{122}}, \; -\frac{5}{\sqrt{122}}, \; \frac{9}{\sqrt{122}} \right\rangle$,
$\mathbf{T}(1) = \mathbf{T}(t)$

5. $\mathbf{r}'(t) = \langle -\pi\sin\pi t, \pi\cos\pi t, 1 \rangle$,
$\mathbf{T}(t) = \frac{1}{\sqrt{\pi^2+1}}\langle -\pi\sin\pi t, \; \pi\cos\pi t, \; 1 \rangle$,
$\mathbf{T}(1) = \left\langle 0, \; -\frac{\pi}{\sqrt{\pi^2+1}}, \; \frac{1}{\sqrt{\pi^2+1}} \right\rangle$

7. $\kappa(t) = \frac{e^t}{(1+e^{2t})^{3/2}}$ **9.** $\kappa(t) = 0$ **11.** $\kappa = \frac{2\sqrt{74}}{27}$

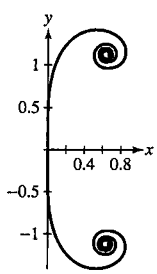
13. $\kappa = \frac{\sqrt{\pi^2+5}}{(\pi^2+1)^{3/2}} \approx 0.108$ **15.** $\kappa(3) = \frac{e^3}{(3^6+1)^{3/2}} \approx 0.0025$

17. $\kappa(2) = \frac{48\sqrt{41}}{210{,}125} \approx 0.0015$

19. $\kappa\left(\frac{\pi}{3}\right) = \frac{\sqrt{330}}{4} \approx 4.54, \kappa\left(\frac{\pi}{2}\right) = \frac{1}{5} = 0.2$ **23.** $\alpha = \pm\sqrt{2}$

29. $\kappa(2) = \frac{3\sqrt{10}}{800} \approx 0.012$ **31.** $\kappa(\pi) = \frac{\pi\sqrt{2}}{4} \approx 1.11$

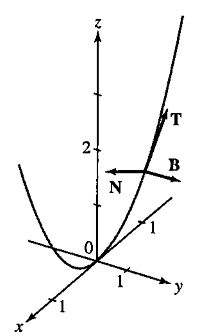
35. $\kappa(t) = t^2$



37. $\mathbf{N}(t) = \langle 0, \; -\sin 2t, \; -\cos 2t \rangle$

39. $\mathbf{N}\left(\frac{\pi}{4}\right) = \left\langle -\frac{\sqrt{2}}{3\sqrt{3}}, \; -\frac{2}{3\sqrt{3}} \right\rangle$, $\mathbf{N}\left(\frac{3\pi}{4}\right) = \left\langle \frac{\sqrt{2}}{3\sqrt{3}}, \; \frac{2}{3\sqrt{3}} \right\rangle$

41.
$\mathbf{T}(1) = \left\langle 0, \frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5} \right\rangle, \mathbf{N}(1) = \left\langle 0, -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle, \mathbf{B}(1) = \langle 1, 0, 0 \rangle$



$\mathbf{T}(1) = \left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right\rangle, \mathbf{N}(1) = \left\langle -\frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle, \mathbf{B}(1) = \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$

45. $\mathbf{N}\left(\pi^{1/3}\right) = \left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$ **47.** $\mathbf{N}(1) = \frac{1}{\sqrt{13}} \langle -3, 2 \rangle$

49. $\mathbf{N}(1) = \frac{1}{\sqrt{2}} \langle 0, 1, -1 \rangle$

51. $\mathbf{N}(0) = \frac{1}{6} \left\langle -\sqrt{6}, 2\sqrt{6}, -\sqrt{6} \right\rangle$

53. **(a)**
$\mathbf{T}(1) = \left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right\rangle, \mathbf{N}(1) = \left\langle -\frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle, \mathbf{B}(1) = \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$
(b) $6x - 6y + 3z = 1$

55. **(a)** $\mathbf{T}(t) = \left\langle \frac{1}{\sqrt{2+4t^2}}, \frac{-1}{\sqrt{2+4t^2}}, \frac{2t}{\sqrt{2+4t^2}} \right\rangle,$
$\mathbf{N}(t) = \left\langle \frac{-t\sqrt{2}}{\sqrt{2+4t^2}}, \frac{t\sqrt{2}}{\sqrt{2+4t^2}}, \frac{\sqrt{2}}{\sqrt{2+4t^2}} \right\rangle$
(b) $\mathbf{B}(t) = \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle$
(c) The osculating planes are all parallel to each other, with equation $x + y = c$ for some c.

59. $\langle \cos t, \sin t \rangle$, that is, the unit circle itself.

61. $\mathbf{c}(t) = \left\langle -4, \frac{7}{2} \right\rangle + \frac{5^{3/2}}{2} \langle \cos t, \sin t \rangle$

63. $\mathbf{c}(t) = \langle \pi, -2 \rangle + 4 \langle \cos t, \sin t \rangle$

65. $\mathbf{c}(t) = \left\langle -1 - 2\cos t, \frac{2\sin t}{\sqrt{2}}, \frac{2\sin t}{\sqrt{2}} \right\rangle$

73. $\kappa(\theta) = 1$ **75.** $\kappa(\theta) = \frac{1}{\sqrt{2}} e^{-\theta}$

91. **(a)** $\mathbf{N}(0) = \left\langle -\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right\rangle$ **(b)** $\mathbf{N}(1) = \frac{1}{\sqrt{66}} \langle 4, 7, -2 \rangle$

Section 13.5 Preliminary Questions

1. No, since the particle may change its direction. **2.** $\mathbf{a}(t)$

3. Statement (a), their velocity vectors point in the same direction.

4. The velocity vector always points in the direction of motion. Since the vector $\mathbf{N}(t)$ is orthogonal to the direction of motion, the vectors $\mathbf{a}(t)$ and $\mathbf{v}(t)$ are orthogonal.

5. Description (b), parallel **6.** $\|\mathbf{a}(t)\| = 8 \text{ cm/s}^2$ **7.** $a_{\mathbf{N}}$

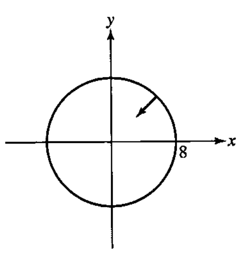
Section 13.5 Exercises

1. $h = -0.2 : \langle -0.085, 1.91, 2.635 \rangle$
$h = -0.1 : \langle -0.19, 2.07, 2.97 \rangle$
$h = 0.1 : \langle -0.41, 2.37, 4.08 \rangle$
$h = 0.2 : \langle -0.525, 2.505, 5.075 \rangle$
$\mathbf{v}(1) \approx \langle -0.3, 2.2, 3.5 \rangle, \ v(1) \approx 4.1$

3. $\mathbf{v}(1) = \langle 3, -1, 8 \rangle, \ \mathbf{a}(1) = \langle 6, 0, 8 \rangle, \ v(1) = \sqrt{74}$

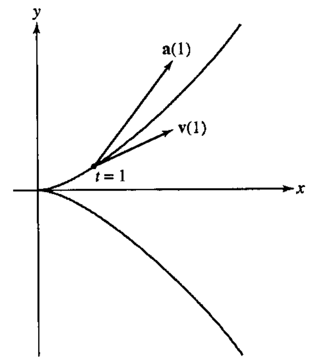
5. $\mathbf{v}(\frac{\pi}{3}) = \left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \right\rangle, \mathbf{a}(\frac{\pi}{3}) = \left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}, 9 \right\rangle, v(\frac{\pi}{3}) = 1$

7. $\mathbf{a}(t) = -2\langle \cos \frac{t}{2}, \sin \frac{t}{2} \rangle; \ \mathbf{a}(\frac{\pi}{4}) \approx \langle -1.85, -0.077 \rangle$



$R(t) = 8\langle \cos \frac{t}{2}, \sin \frac{t}{2} \rangle$

9.



$\mathbf{r}(t) = \langle t^2, t^3 \rangle$

11. $\mathbf{v}(t) = \left\langle \frac{3t^2+2}{6}, 4t - 2 \right\rangle$ **13.** $\mathbf{v}(t) = \mathbf{i} + t\mathbf{k}$

15. $\mathbf{v}(t) = \left\langle \frac{t^2}{2} + 3, 4t - 2 \right\rangle, \ \mathbf{r}(t) = \left\langle \frac{t^3}{6} + 3t, 2t^2 - 2t \right\rangle$

17. $\mathbf{v}(t) = \mathbf{i} + \frac{t^2}{2}\mathbf{k}, \ \mathbf{r}(t) = t\mathbf{i} + \mathbf{j} + \frac{t^3}{6}\mathbf{k}$

19. $v_0 = \sqrt{5292} \approx 72.746$ m/s **23.** $H = 355$ m

25. **(a)** Assume that $r(0) = \langle 150, 75, 5 \rangle$
$$a(t) = \langle 0, 0, -32 \rangle, \ v(t) = \langle 40, 35, -32t + 32 \rangle$$
$$r(t) = \left\langle 40t + 150, 35t + 75, -16t^2 + 32t + 5 \right\rangle$$
(b) $z = 5$ when $t = 0$ or 2. At $t = 2$, $r(2) = \langle 230, 145, 5 \rangle$, so the player is in bounds, since $(300, 150, z)$ is the maximum possible point to be in bounds.

27. $\mathbf{r}(10) = \langle 45, -20 \rangle$

29. **(a)** At its original position **(b)** No

31. The speed is decreasing.

33. $a_{\mathbf{T}} = 0, \ a_{\mathbf{N}} = 1$ **35.** $a_{\mathbf{T}} = \frac{7}{\sqrt{6}}, \ a_{\mathbf{N}} = \sqrt{\frac{53}{6}}$

37. $\mathbf{a}(-1) = -\frac{2}{\sqrt{10}}\mathbf{T} + \frac{6}{\sqrt{10}}\mathbf{N}$ with $\mathbf{T} = \frac{1}{\sqrt{10}} \langle 1, -3 \rangle$ and $\mathbf{N} = \frac{1}{\sqrt{10}} \langle -3, -1 \rangle$

39. $a_{\mathbf{T}}(4) = 4, \ a_{\mathbf{N}}(4) = 1$, so $\mathbf{a} = 4\mathbf{T} + \mathbf{N}$, with $\mathbf{T} = \left\langle \frac{1}{9}, \frac{4}{9}, \frac{8}{9} \right\rangle$ and $\mathbf{N} = \left\langle -\frac{4}{9}, -\frac{7}{9}, \frac{4}{9} \right\rangle$

41. $\mathbf{a}(0) = \sqrt{3}\mathbf{T} + \sqrt{2}\mathbf{N}$, with $\mathbf{T} = \frac{1}{\sqrt{3}} \langle 1, 1, 1 \rangle$ and $\mathbf{N} = \frac{1}{\sqrt{2}} \langle -1, 0, 1 \rangle$

43. $\mathbf{a}(\frac{\pi}{2}) = -\frac{\pi}{2\sqrt{3}}\mathbf{T} + \frac{\pi}{\sqrt{6}}\mathbf{N}$, with $\mathbf{T} = \frac{1}{\sqrt{3}} \langle 1, -1, 1 \rangle$ and $\mathbf{N} = \frac{1}{\sqrt{6}} \langle 1, -1, -2 \rangle$

45. $a_{\mathbf{T}} = 0, \ a_{\mathbf{N}} = 0.25 \text{ cm/s}^2$

47. The tangential acceleration is $\frac{50}{\sqrt{2}} \approx 35.36 \text{ m/min}^2$, $v = \sqrt{35.36(30)} \approx 32.56$ m/min

49. $\|\mathbf{a}\| = 1.157 \times 10^5 \text{ km/h}^2$ **51.** $\mathbf{a} = \left\langle -\frac{1}{6}, -1, \frac{1}{6} \right\rangle$

53. (A) slowing down, (B) speeding up, (C) slowing down

57. After 139.91 s, the car will begin to skid. **61.** $R \approx 105$ m

Section 13.6 Preliminary Questions

1. $\frac{dA}{dt} = \frac{1}{2} \|\mathbf{J}\|$ **3.** The period is increased eight-fold.

Section 13.6 Exercises

1. The data support Kepler's prediction;
$T \approx \sqrt{a^3 \cdot 3 \cdot 10^{-4}} \approx 11.9$ years

3. $M \approx 1.897 \times 10^{27}$ kg **5.** $M \approx 2.6225 \times 10^{41}$ kg

11. The satellite's orbit is in the plane $20x - 29y = 9z = 0$.

Chapter 13 Review

1. (a) $-1 < t < 0$ or $0 < t \le 1$ (b) $0 < t \le 2$

3. $\mathbf{r}(t) = \left\langle t^2,\ t,\ \sqrt[3]{3-t^4}\right\rangle, \quad -\infty < t < \infty$

5. $\mathbf{r}'(t) = \left\langle -1,\ -2t^{-3},\ \frac{1}{t}\right\rangle$ **7.** $\mathbf{r}'(0) = \langle 2,\ 0,\ 6\rangle$

9. $\frac{d}{dt}e^t\left\langle 1,\ t,\ t^2\right\rangle = e^t\left\langle 1,\ 1+t,\ 2t+t^2\right\rangle$

11. $\frac{d}{dt}\left(6\mathbf{r}_1(t) - 4\mathbf{r}_2(t)\right)|_{t=3} = \langle 0,\ -8,\ -10\rangle$

13. $\frac{d}{dt}\left(\mathbf{r}_1(t)\cdot\mathbf{r}_2(t)\right)|_{t=3} = 2$

15. $\int_0^3\left\langle 4t+3,\ t^2,\ -4t^3\right\rangle\,dt = \langle 27,\ 9,\ -81\rangle$

17. $\left(3,\ 3,\ \frac{16}{3}\right)$ **19.** $\mathbf{r}(t) = \left\langle 2t^2 - \frac{8}{3}t^3 + t,\ t^4 - \frac{1}{6}t^3 + 1\right\rangle$

21. $L = 2\sqrt{13}$ **23.** $\left\langle 5\cos\frac{2\pi s}{5\sqrt{1+4\pi^2}},\ 5\sin\frac{2\pi s}{5\sqrt{1+4\pi^2}},\ \frac{s}{\sqrt{1+4\pi^2}}\right\rangle$

25. $v_0 \approx 67.279$ m/s **27.** $\left(0,\ \frac{11}{2},\ 38\right)$

29. $\mathbf{T}(\pi) = \left\langle \frac{-1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ 0\right\rangle$ **31.** $\kappa(1) = \frac{1}{2^{3/2}}$

33. $\mathbf{a} = \frac{1}{\sqrt{2}}\mathbf{T} + 4\mathbf{N}$, where $\mathbf{T} = \langle -1,\ 0\rangle$ and $\mathbf{N} = \langle 0,\ -1\rangle$

35. $\kappa = \frac{13}{16}$ **37.** $\mathbf{c}(t) = \left\langle \frac{25}{2} + \frac{17^{3/2}}{2}\cos t, -32 + \frac{17^{3/2}}{2}\sin t\right\rangle$

39. $2x - 4y + 2z = -3$

Chapter 14

Section 14.1 Preliminary Questions

1. Same shape, but located in parallel planes

2. The parabola $z = x^2$ in the xz-plane **3.** Not possible

4. The vertical lines $x = c$ with distance of 1 unit between adjacent lines

5. In the contour map of $g(x,\ y) = 2x$, the distance between two adjacent vertical lines is $\frac{1}{2}$.

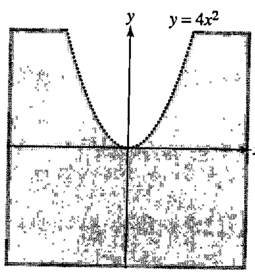
Section 14.1 Exercises

1. $f(2,\ 2) = 18,\ f(-1,\ 4) = -5$

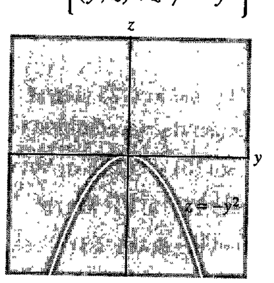
3. $h(3,\ 8,\ 2) = 6; h(3,\ -2,\ -6) = -\frac{1}{6}$

5. The domain is the entire xy-plane.

7.

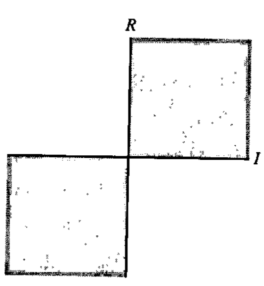


9. $\mathcal{D} = \left\{(y, z) : z \neq -y^2\right\}$



$z + y^2 \neq 0$

11.



$IR \ge 0$

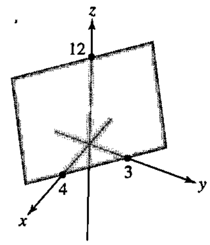
13. Domain: entire $(x,\ y,\ z)$−space; range: entire real line

15. Domain: $\{(r,\ s,\ t) : |rst| \le 4\}$; range: $\{w : 0 \le w \le 4\}$

17. $f \leftrightarrow$ (B), $g \leftrightarrow$ (A)

19. (a) D (b) C (c) E (d) B (e) A (f) F

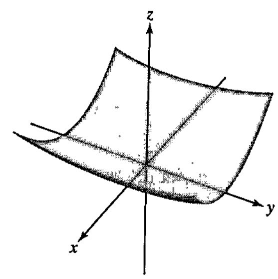
21.



Horizontal trace: $3x + 4y = 12 - c$ in the plane $z = c$

Vertical trace: $z = (12 - 3a) - 4y$ and $z = -3x + (12 - 4a)$ in the planes $x = a$, and $y = a$, respectively

23.

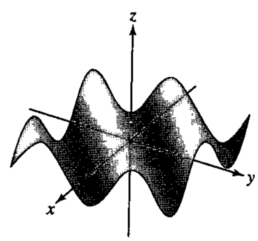


The horizontal traces are ellipses for $c > 0$.

The vertical trace in the plane $x = a$ is the parabola $z = a^2 + 4y^2$.

The vertical trace in the plane $y = a$ is the parabola $z = x^2 + 4a^2$.

25.

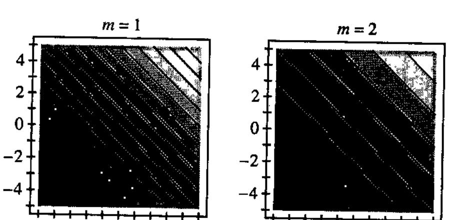


The horizontal traces in the plane $z = c$, $|c| \le 1$, are the lines $x - y = \sin^{-1}c + 2k\pi$ and $x - y = \pi - \sin^{-1}c + 2k\pi$, for integer k.

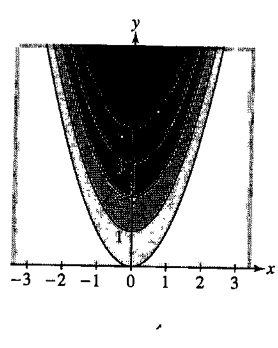
The vertical trace in the plane $x = a$ is $z = \sin(a - y)$.

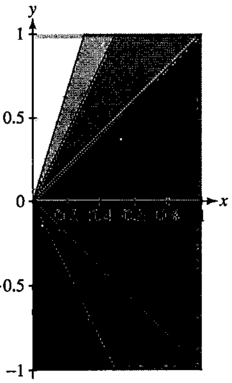
The vertical trace in the plane $y = a$ is $z = \sin(x - a)$.

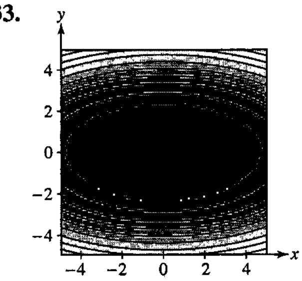
27. $m = 1 : m = 2 :$

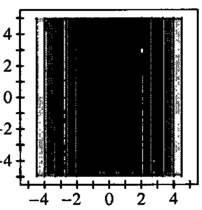


29.



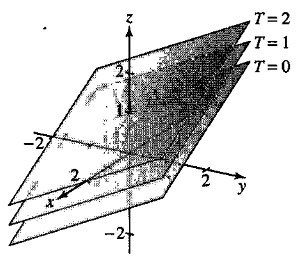


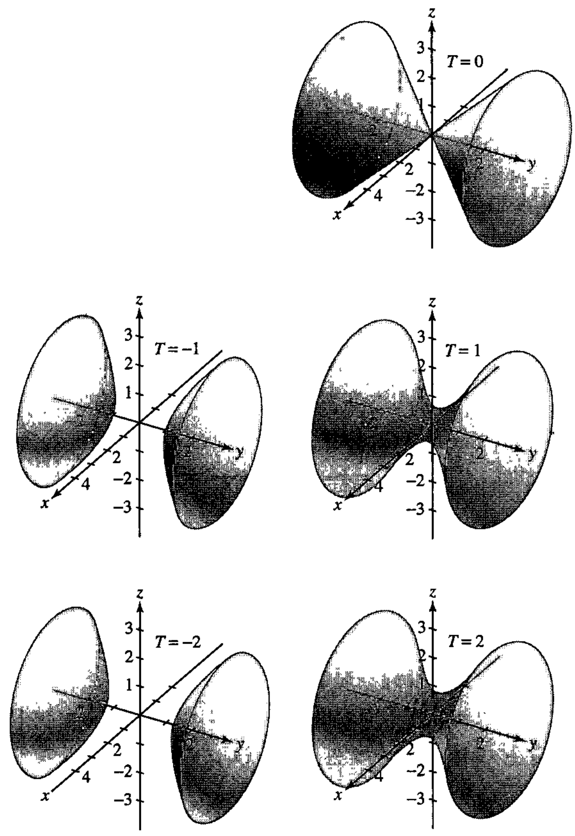
33.


35.


37. $m = 6: \ f(x, y) = 2x + 6y + 6$
$m = 3: \ f(x, y) = x + 3y + 3$

39. **(a)** Only at (A) **(b)** Only at (C) **(c)** West

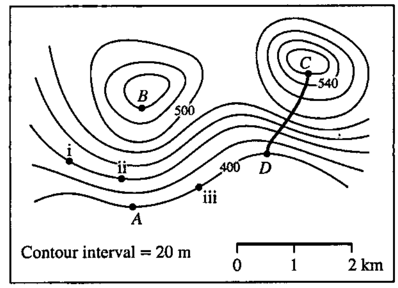
41.


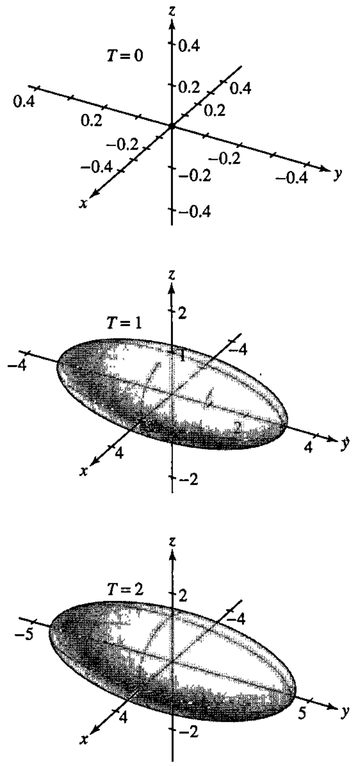
43.


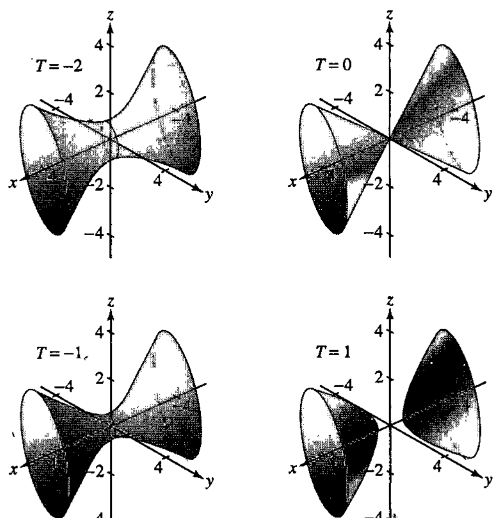
45. Average ROC from B to $C = 0.000625 \ \text{kg/m}^3 \cdot \text{ppt}$

47. At point A

49. Average ROC from A to $B \approx 0.0737$, average ROC from A to $C \approx 0.0457$

51.


53.


55.


57. $f(r, \theta) = \cos\theta$; the level curves are
$\theta = \pm\cos^{-1}(c)$ for $|c| < 1, \ \ c \neq 0$;
the y-axis for $c = 0$;
the positive x-axis for $c = 1$;
the negative x-axis for $c = -1$.

Section 14.2 Preliminary Questions

1. $D^*(p, r)$ consists of all points in $D(p, r)$ other than p itself.
2. $f(2, 3) = 27$ **3.** All three statements are true.
4. $\lim_{(x, y)\to(0, 0)} f(x, y)$ does not exist.

Section 14.2 Exercises

1. $\lim_{(x, y)\to(1, 2)} (x^2 + y) = 3$ **3.** $\lim_{(x, y)\to(2, -1)} (xy - 3x^2y^3) = 10$
5. $\lim_{(x, y)\to(\frac{\pi}{4}, 0)} \tan x \cos y = 1$
7. $\lim_{(x, y)\to(1, 1)} \frac{e^{x^2} - e^{-y^2}}{x+y} = \frac{1}{2}(e - e^{-1})$
9. $\lim_{(x, y)\to(2, 5)} (g(x, y) - 2f(x, y)) = 1$
11. $\lim_{(x, y)\to(2, 5)} e^{f(x, y)^2 - g(x, y)} = e^2$
13. No; the limit along the x-axis and the limit along the y-axis are different.
15. The limit is $\frac{1+m^3}{m^2}$ for all $m \neq 0$.
17. The limit along the x-axis is $\lim_{(x,y)\to(0,0)} \frac{x}{x^2+y^2} = \lim_{x\to 0} \frac{1}{x}$, which does not exist.
19. $\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{\sqrt{x^2 + y^2}} = 0$
21. The limit does not exist, because the function values as (x, y) approach $(0, 0)$ along the line $y = mx$ depend on the value of m.

$$\lim_{(x,y)\to(0,0) y=mx} \frac{xy}{3x^2 + 2y^2} = \lim_{x\to 0} \frac{mx^2}{3x^2 + 2m^2x^2} = \frac{m}{2m^2 + 3}$$

23. Along the x-coordinate axis ($y = z = 0$),

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x + y + z}{x^2 + y^2 + z^2} = \lim_{(x,y,z)\to(0,0,0)} \frac{1}{x} = \infty$$

25. $\lim_{(x, y)\to(4, 0)} (x^2 - 16) \cos\left(\frac{1}{(x - 4)^2 + y^2}\right) = 0$
27. $\lim_{(z, w)\to(-2, 1)} \frac{z^4 \cos(\pi w)}{e^{z+w}} = -16e$
29. $\lim_{(x, y)\to(4, 2)} \frac{y-2}{\sqrt{x^2 - 4}} = 0$ **31.** $\lim_{(x, y)\to(3, 4)} \frac{1}{\sqrt{x^2 + y^2}} = \frac{1}{5}$
33. $\lim_{(x, y)\to(1, -3)} e^{x-y} \ln(x - y) = e^4 \ln(4)$
35. $\lim_{(x, y)\to(-3, -2)} (x^2y^3 + 4xy) = -48$
37. $\lim_{(x, y)\to(0, 0)} \tan(x^2 + y^2)\tan^{-1}\left(\frac{1}{x^2+y^2}\right) = 0$
39. $\lim_{(x, y)\to(0, 0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} = 2$
43. $\lim_{(x, y)\to Q} g(x, y) = 4$ **45.** Yes

Section 14.3 Preliminary Questions

1. $\frac{\partial}{\partial x}(x^2y^2) = 2xy^2$
2. In this case, the Constant Multiple Rule can be used. In the second part, since y appears in both the numerator and the denominator, the Quotient Rule is preferred.
3. (a), (c) **4.** $f_x = 0$ **5.** (a), (d)

Section 14.3 Exercises

3. $\frac{\partial}{\partial y} \frac{y}{x+y} = \frac{x}{(x+y)^2}$ **5.** $f_z(2, 3, 1) = 6$
7. $m = 10$ **9.** $f_x(A) \approx 10$, $f_y(A) \approx -20$ **11.** NW
13. $\frac{\partial}{\partial x}(x^2 + y^2) = 2x$, $\frac{\partial}{\partial y}(x^2 + y^2) = 2y$
15. $\frac{\partial}{\partial x}(x^4y + xy^{-2}) = 4x^3y + y^{-2}$, $\frac{\partial}{\partial y}(x^4y + xy^{-2}) = x^4 - 2xy^{-3}$
17. $\frac{\partial}{\partial x}\left(\frac{x}{y}\right) = \frac{1}{y}$, $\frac{\partial}{\partial y}\left(\frac{x}{y}\right) = \frac{-x}{y^2}$
19. $\frac{\partial}{\partial x}\left(\sqrt{9 - x^2 - y^2}\right) = \frac{-x}{\sqrt{9-x^2-y^2}}$, $\frac{\partial}{\partial y}\left(\sqrt{9 - x^2 - y^2}\right) = \frac{-y}{\sqrt{9-x^2-y^2}}$
21. $\frac{\partial}{\partial x}(\sin x \sin y) = \sin y \cos x$, $\frac{\partial}{\partial y}(\sin x \sin y) = \sin x \cos y$
23. $\frac{\partial}{\partial x}\left(\tan \frac{x}{y}\right) = \frac{1}{y}\sec^2 \frac{x}{y}$, $\frac{\partial}{\partial y}\left(\tan \frac{x}{y}\right) = -\frac{x}{y^2}\sec^2 \frac{x}{y}$
25. $\frac{\partial}{\partial x} \ln(x^2 + y^2) = \frac{2x}{x^2+y^2}$, $\frac{\partial}{\partial y} \ln(x^2 + y^2) = \frac{2y}{x^2+y^2}$
27. $\frac{\partial}{\partial r} e^{r+s} = e^{r+s}$, $\frac{\partial}{\partial s} e^{r+s} = e^{r+s}$
29. $\frac{\partial}{\partial x} e^{xy} = ye^{xy}$, $\frac{\partial}{\partial y} e^{xy} = xe^{xy}$
31. $\frac{\partial z}{\partial y} = -2xe^{-x^2-y^2}$, $\frac{\partial z}{\partial y} = -2ye^{-x^2-y^2}$
33. $\frac{\partial U}{\partial t} = -e^{-rt}$, $\frac{\partial U}{\partial r} = \frac{-e^{-rt}(rt+1)}{r^2}$
35. $\frac{\partial}{\partial x} \sinh(x^2y) = 2xy \cosh(x^2y)$, $\frac{\partial}{\partial y} \sinh(x^2y) = x^2 \cosh(x^2y)$
37. $\frac{\partial w}{\partial x} = y^2z^3$, $\frac{\partial w}{\partial y} = 2xz^3y$, $\frac{\partial w}{\partial z} = 3xy^2z^2$
39. $\frac{\partial Q}{\partial L} = \frac{M - Lt}{M^2} e^{-Lt/M}$, $\frac{\partial Q}{\partial M} = \frac{L(Lt - M)}{M^3} e^{-Lt/M}$, $\frac{\partial Q}{\partial t} = -\frac{L^2}{M^2} e^{-Lt/M}$
41. $f_x(1, 2) = -164$ **43.** $g_u(1, 2) = \ln 3 + \frac{1}{3}$
45. $N = 2865.058$, $\Delta N \approx -217.74$
47. (a) $I(95, 50) \approx 73.1913$ (b) $\frac{\partial I}{\partial T}$; 1.66
49. A 1-cm increase in r
51. $\frac{\partial W}{\partial E} = -\frac{1}{kT} e^{-E/kT}$, $\frac{\partial W}{\partial T} = \frac{E}{kT^2} e^{-E/kT}$
55. (a), (b) **57.** $\frac{\partial^2 f}{\partial x^2} = 6y$, $\frac{\partial^2 f}{\partial y^2} = -72xy^2$
59. $h_{vv} = \frac{32u}{(u + 4v)^3}$ **61.** $f_{yy}(2, 3) = -\frac{4}{9}$
63. $f_{xyxzy} = 0$ **65.** $f_{uuv} = 2v \sin(u + v^2)$
67. $F_{rst} = 0$ **69.** $F_{uu\theta} = \cosh(uv + \theta^2) \cdot 2\theta v^2$
71. $g_{xyz} = \frac{3xyz}{(x^2 + y^2 + z^2)^{5/2}}$ **73.** $f(x, y) = x^2y$ **77.** $B = A^2$

Section 14.4 Preliminary Questions

1. $L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$
2. f is locally linear at (a, b) if $f(x, y) = L(x, y) + e(x, y)$, where $e(x, y)$ satisfies $\lim_{(x,y)\to(a,b)} \frac{e(x,y)}{\sqrt{(x-a)^2+(y-b)^2}} = 0$
3. (b) **4.** $f(2, 3.1) \approx 8.7$ **5.** $\Delta f \approx -0.1$
6. Criterion for Differentiability

Section 14.4 Exercises

1. $z = -34 - 20x + 16y$ **3.** $z = 5x + 10y - 14$
5. $z = 8x - 2y - 13$ **7.** $z = 4r - 5s + 2$
9. $z = \left(\frac{4}{5} + \frac{12}{25} \ln 2\right) - \frac{12}{25}x + \frac{12}{25}y$ **11.** $\left(-\frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right)$
13. (a) $f(x, y) = -16 + 4x + 12y$
(b) $f(2.01, 1.02) \approx 4.28$; $f(1.97, 1.01) \approx 4$
15. $\Delta f \approx 3.56$ **17.** $f(0.01, -0.02) \approx 0.98$

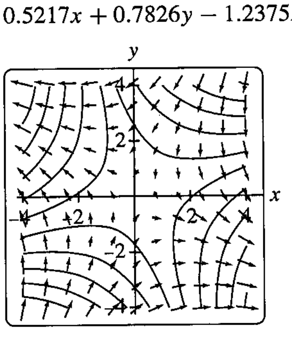
. $L(x, y, z) = \frac{5}{12}\sqrt{3}x + \frac{5}{12}\sqrt{3}y + 2\sqrt{3}z - 5\sqrt{3}$
1. 5.07 **23.** 8.44 **25.** 4.998 **27.** 3.945
29. $f(-2.1, 3.1) \approx 4.2$ **31.** $\Delta I \approx 0.5644$
33. **(b)** $\Delta H \approx 0.022$ m
35. **(b)** 6% **(c)** 1% error in r
37. **(a)** \$7.10 **(b)** \$28.85, \$57.69 **(c)** −\$74.24
39. Maximum error in V is about 8.948 m.

Section 14.5 Preliminary Questions

1. **(b)** $\langle 3, 4\rangle$ **2.** False
3. ∇f points in the direction of maximum rate of increase of f and is normal to the level curve of f.
4. **(b)** NW and **(c)** SE **5.** $3\sqrt{2}$

Section 14.5 Exercises

1. (a) $\nabla f = \langle y^2, 2xy\rangle$, $\mathbf{r}'(t) = \langle t, 3t^2\rangle$
(b) $\frac{d}{dt}(f(\mathbf{r}(t)))\Big|_{t=1} = 4$; $\frac{d}{dt}(f(\mathbf{r}(t)))\Big|_{t=-1} = -4$
3. A: zero, B: negative, C: positive, D: zero
5. $\nabla f = -\sin(x^2 + y)\langle 2x, 1\rangle$
7. $\nabla h = \langle yz^{-3}, xz^{-3}, -3xyz^{-4}\rangle$
9. $\frac{d}{dt}(f(\mathbf{r}(t)))\Big|_{t=0} = -7$ **11.** $\frac{d}{dt}(f(\mathbf{r}(t)))\Big|_{t=0} = -3$
13. $\frac{d}{dt}(f(\mathbf{r}(t)))\Big|_{t=0} = 5\cos 1 \approx 2.702$
15. $\frac{d}{dt}(f(\mathbf{r}(t)))\Big|_{t=4} = -56$
17. $\frac{d}{dt}(f(\mathbf{r}(t)))\Big|_{t=\pi/4} = -1 + \frac{\pi}{8} \approx 1.546$
19. $\frac{d}{dt}(g(\mathbf{r}(t)))\Big|_{t=1} = 0$
21. $D_{\mathbf{u}}f(1, 2) = 8.8$ **23.** $D_{\mathbf{u}}f\left(\frac{1}{6}, 3\right) = \frac{39}{4\sqrt{2}}$
25. $D_{\mathbf{u}}f(3, 4) = \frac{7\sqrt{2}}{290}$ **27.** $D_{\mathbf{u}}f(1, 0) = \frac{6}{\sqrt{13}}$
29. $D_{\mathbf{u}}f(1, 2, 0) = -\frac{1}{\sqrt{3}}$ **31.** $D_{\mathbf{u}}f(3, 2) = \frac{-50}{\sqrt{13}}$
33. $D_{\mathbf{u}}f(P) = -\frac{e^5}{3} \approx -49.47$
35. **(a)** $m = 3$, angle of inclination is approximately $1.249 \approx 71.6°$.
(b) $m = 4\sqrt{2} \approx 5.66$, angle of inclination is approximately $1.396 \approx 80.0°$.
(c) $m = -4\sqrt{2} \approx -5.66$, angle of inclination is approximately $-1.396 \approx -80.0°$.
(d) $m = \sqrt{34} \approx 5.83$, angle from due East is approximately 121°.
37. f is increasing at P in the direction of v.
39. $D_{\mathbf{u}}f(P) = \frac{\sqrt{6}}{2}$ **41.** $\langle 6, 2, -4\rangle$
43. $\left(\frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, -\frac{2}{\sqrt{17}}\right)$ and $\left(-\frac{4}{\sqrt{17}}, -\frac{9}{\sqrt{17}}, \frac{2}{\sqrt{17}}\right)$
45. $9x + 10y + 5z = 33$
47. $0.5217x + 0.7826y - 1.2375z = -5.309$
49.



51. $f(x, y, z) = x^2 + y + 2z$
53. $f(x, y, z) = xz + y^2$ **57.** $\Delta f \approx 0.08$
59. **(a)** $\langle 34, 18, 0\rangle$
(b) $\left\langle 2 + \frac{32}{\sqrt{21}}t, 2 + \frac{16}{\sqrt{21}}t, 8 - \frac{8}{\sqrt{21}}t\right\rangle$; ≈ 4.58 s
63. $x = 1 - 4t$, $y = 2 + 26t$, $z = 1 - 25t$
75. $y = \sqrt{1 - \ln(\cos^2 x)}$

Section 14.6 Preliminary Questions

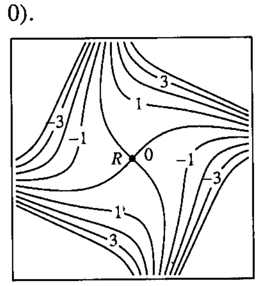
1. **(a)** $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ **(b)** u and v
2. (a) **3.** $f(u, v)|_{(r,s)=(1,1)} = e^2$ **4.** (b) **5.** (c) **6.** No

Section 14.6 Exercises

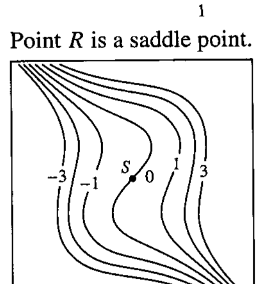
1. **(a)** $\frac{\partial f}{\partial x} = 2xy^3$, $\frac{\partial f}{\partial y} = 3x^2y^2$, $\frac{\partial f}{\partial z} = 4z^3$
(b) $\frac{\partial x}{\partial s} = 2s$, $\frac{\partial y}{\partial s} = t^2$, $\frac{\partial z}{\partial s} = 2st$
(c) $\frac{\partial f}{\partial s} = 7s^6t^6 + 8s^7t^4$
3. $\frac{\partial f}{\partial s} = 6rs^2$, $\frac{\partial f}{\partial r} = 2s^3 + 4r^3$
5. $\frac{\partial g}{\partial u} = -10\sin(10u - 20v)$, $\frac{\partial g}{\partial v} = 20\sin(10u - 20v)$
7. $\frac{\partial F}{\partial y} = xe^{x^2+xy}$ **9.** $\frac{\partial h}{\partial t_2} = 0$
11. $\frac{\partial f}{\partial u}\Big|_{(u,v)=(-1,-1)} = 1$, $\frac{\partial f}{\partial v}\Big|_{(u,v)=(-1,-1)} = -2$
13. $\frac{\partial g}{\partial \theta}\Big|_{(r,\theta)=(2\sqrt{2}, \pi/4)} = -\frac{1}{6}$ **15.** $\frac{\partial g}{\partial u}\Big|_{(u,v)=(0,1)} = 2\cos 2$
17. −26.8 ft/s **19.** $4\pi^3 - 3\pi^2 - 1$
23. **(a)** $F_x = z^2 + y$, $F_y = 2yz + x$, $F_z = 2xz + y^2$
(b) $\frac{\partial z}{\partial x} = -\frac{z^2 + y}{2xz + y^2}$, $\frac{\partial z}{\partial y} = -\frac{2yz + x}{2xz + y^2}$
27. $\frac{\partial z}{\partial x} = -\frac{2xy + z^2}{2xz + y^2}$ **29.** $\frac{\partial z}{\partial y} = -\frac{xe^{xy} + 1}{x\cos(xz)}$
31. $\frac{\partial w}{\partial y} = \frac{-y(w^2 + x^2)^2}{w\left((w^2 + y^2)^2 + (w^2 + x^2)^2\right)}$; at (1, 1, 1), $\frac{\partial w}{\partial y} = -\frac{1}{2}$
35. $\nabla\left(\frac{1}{r}\right) = -\frac{1}{r^3}\mathbf{r}$ **37.** (c) $\frac{\partial z}{\partial x} = \frac{x - 6}{z + 4}$
39. $\frac{\partial P}{\partial T} = -\frac{F_T}{F_P} = -\frac{-nR}{V-nb} = \frac{nR}{V-nb}$

Section 14.7 Preliminary Questions

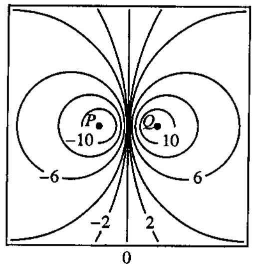
1. f has a local (and global) min at (0, 0); g has a saddle point at (0, 0).
2.



Point R is a saddle point.



Point S is neither a local extremum nor a saddle point.



Point P is a local minimum and point Q is a local maximum.

3. Statement (a)

Section 14.7 Exercises

1. **(b)** $P_1 = (0, 0)$ is a saddle point, $P_2 = \left(2\sqrt{2}, \sqrt{2}\right)$ and $P_3 = \left(-2\sqrt{2}, -\sqrt{2}\right)$ are local minima; absolute minimum value of f is -4.

3. $(0, 0)$ saddle point, $\left(\frac{13}{64}, -\frac{13}{32}\right)$ and $\left(-\frac{1}{4}, \frac{1}{2}\right)$ local minima

5. **(c)** $(0, 0)$, $(1, 0)$, and $(0, -1)$ saddle points, $\left(\frac{1}{3}, -\frac{1}{3}\right)$ local minimum.

7. $\left(-\frac{2}{3}, -\frac{1}{3}\right)$ local minimum

9. $(-2, -1)$ local maximum, $\left(\frac{5}{3}, \frac{5}{6}\right)$ saddle point

11. $\left(0, \pm\sqrt{2}\right)$ saddle points, $\left(\frac{2}{3}, 0\right)$ local maximum, $\left(-\frac{2}{3}, 0\right)$ local minimum

13. $(0, 0)$ saddle point, $(1, 1)$ and $(-1, -1)$ local minima

15. $(0, 0)$ saddle point, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ local maximum, $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ local minimum

17. Critical points are $\left(j\pi, k\pi + \frac{\pi}{2}\right)$, for
j, k even: saddle points
j, k odd: local maxima
j even, k odd: local minima
j odd, k even: saddle points

19. $\left(1, \frac{1}{2}\right)$ local maximum **21.** $\left(\frac{3}{2}, -\frac{1}{2}\right)$ saddle point

23. $\left(-\frac{1}{6}, -\frac{17}{18}\right)$ local minimum

27. $x = y = 0.27788$ local minimum

29. Global maximum 2, global minimum 0

31. Global maximum 1, global minimum $\frac{1}{35}$

35. Maximum value $\frac{1}{3}$

37. Global minimum $f(0, 1) = -2$, global maximum $f(1, 0) = 1$

39. Global minimum, $f(0, 0) = 0$, global maximum $f(1, 1) = 3$

41. Global minimum, $f(0, 0) = 0$, global maximum $f(1, 0) = f(0, 1) = 1$

43. Global minimum, $f(1, 2) = -5$, global maximum $f(0, 0) = f(3, 3) = 0$

45. Global minimum, $f(-0.4343, 0.9) = f(-0.4343, -0.9) \approx -0.5161$, global maximum $f(0.7676, 0.6409) = f(0.7676, -0.6409) \approx 1.2199$

47. Maximum volume $\frac{3}{4}$

49. **(a)** No. In the box B with minimal surface area, z is smaller than $\sqrt[3]{V}$, which is the side of a cube with volume V.

(b) Width: $x = (2V)^{1/3}$; length: $y = (2V)^{1/3}$; height: $z = \left(\frac{V}{4}\right)^{1/3}$

53. The fence should be cut into 12 pieces 10 m long, forming three 10 × 10 squares.

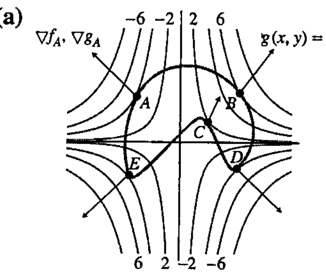
55. $V = \frac{64{,}000}{\pi} \approx 20{,}372\ \text{cm}^3$ **57.** $f(x) = 1.9629x - 1.5519$

Section 14.8 Preliminary Questions

1. Statement (b)

2. f had a local maximum 2, under the constraint, at A; $f(B)$ is neither a local minimum nor a local maximum of f.

3. **(a)**



Contour plot of $f(x, y)$
(contour interval 2)

(b) Global minimum -4, global maximum 6

Section 14.8 Exercises

1. **(c)** Critical points $(-1, -2)$ and $(1, 2)$

(d) Maximum 10, minimum -10

3. Maximum $4\sqrt{2}$, minimum $-4\sqrt{2}$

5. Minimum $\frac{36}{13}$, no maximum value

7. Maximum $\frac{8}{3}$, minimum $-\frac{8}{3}$

9. Maximum $\sqrt{2}$, minimum 1

11. Maximum 3.7, minimum -3.7

13. Maxima at $f(\pm 4, \pm 4, 2) = 20$, and minima at $f(\pm 4, \mp 4, -2) = -20$

15. Maximum $2\sqrt{2}$, minimum $-2\sqrt{2}$ **17.** $(-1, e^{-1})$

19. **(a)** $\frac{h}{r} = \sqrt{2}$ **(b)** $\frac{h}{r} = \sqrt{2}$

(c) There is no cone of fixed V with maximal S.

21. $(8, -2)$ **23.** $\left(\frac{48}{97}, \frac{108}{97}\right)$ **25.** $\frac{a^a b^b}{(a+b)^{a+b}}$ **27.** $\sqrt{\frac{a^a b^b}{(a+b)^{a+b}}}$

33. $r = 3$, $h = 6$ **35.** $x + y + z = 3$

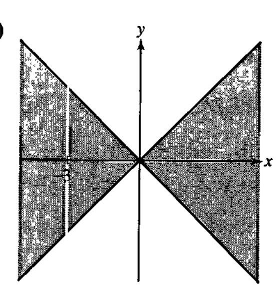
41. $\frac{25}{3}$ **43.** $\left(\frac{-6}{\sqrt{105}}, \frac{-3}{\sqrt{105}}, \frac{30}{\sqrt{105}}\right)$ **45.** $(-1, 0, 2)$

47. Minimum $\frac{138}{11} \approx 12.545$; no maximum value

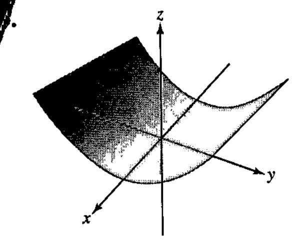
51. (b) $\lambda = \frac{c}{2p_1 p_2}$

Chapter 14 Review

1. **(a)**



(b) $f(3, 1) = \frac{\sqrt{2}}{3}$, $f(-5, -3) = -2$ **(c)** $\left(-\frac{5}{3}, 1\right)$



Vertical and horizontal traces: the line $z = (c^2 + 1) - y$ in the plane $x = c$, the parabola $z = x^2 - c + 1$ in the plane $y = c$.

5. **(a)** Graph (B) **(b)** Graph (C) **(c)** Graph (D) **(d)** Graph (A)

7. **(a)** Parallel lines $4x - y = \ln c$, $c > 0$, in the xy-plane
(b) Parallel lines $4x - y = e^c$ in the xy-plane
(c) Hyperbolas $3x^2 - 4y^2 = c$ in the xy-plane
(d) Parabolas $x = c - y^2$ in the xy-plane

9. $\lim_{(x,y)\to(1,-3)} (xy + y^2) = 6$

11. The limit does not exist.

13. $\lim_{(x,y)\to(1,-3)} (2x + y)e^{-x+y} = -e^{-4}$

17. $f_x = 2,\ f_y = 2y$

19. $f_x = e^{-x-y}(y\cos(xy) - \sin(xy))$
$f_y = e^{-x-y}(x\cos(yx) - \sin(yx))$

21. $f_{xxyz} = -\cos(x + z)$ **23.** $z = 33x + 8y - 42$

25. Estimate, 12.146; calculator value to three places, 11.996.

27. Statements (ii) and (iv) are true.

29. $\frac{d}{dt}\left(f(\mathbf{c}(t))\right)\Big|_{t=2} = 3 + 4e^4 \approx 221.4$

31. $\frac{d}{dt}\left(f(\mathbf{c}(t))\right)\Big|_{t=1} = 4e - e^{3e} \approx -3469.3$

33. $D_{\mathbf{u}}f(3, -1) = -\frac{54}{\sqrt{5}}$

35. $D_{\mathbf{u}}f(P) = -\frac{\sqrt{2}e}{5}$ **37.** $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\rangle$

41. $\frac{\partial f}{\partial s} = 3s^2t + 4st^2 + t^3 - 2st^3 + 6s^2t^2$
$\frac{\partial f}{\partial t} = 4s^2t + 3st^2 + s^3 + 4s^3t - 3s^2t^2$

45. $\frac{\partial z}{\partial x} = -\frac{e^z - 1}{xe^z + e^y}$

47. $(0, 0)$ saddle point, $(1, 1)$ and $(-1, -1)$ local minima

49. $\left(\frac{1}{2}, \frac{1}{2}\right)$ saddle point

53. Global maximum $f(2, 4) = 10$, global minimum $f(-2, 4) = -18$

55. Maximum $\frac{26}{\sqrt{13}}$, minimum $-\frac{26}{\sqrt{13}}$

57. Maximum $\frac{12}{\sqrt{3}}$, minimum $-\frac{12}{\sqrt{3}}$

59. Minimum $= f\left(\frac{1+\sqrt{3}}{3}, \frac{1}{3}, \frac{1-\sqrt{3}}{3}\right) = \frac{6-2\sqrt{3}}{3}$
Maximum $= f\left(\frac{1-\sqrt{3}}{3}, \frac{1}{3}, \frac{1+\sqrt{3}}{3}\right) = \frac{6+2\sqrt{3}}{3}$

61. $r = h = \sqrt[3]{\frac{V}{\pi}},\ S = 3\pi\left(\frac{V}{\pi}\right)^{2/3}$

Chapter 15

Section 15.1 Preliminary Questions

1. $\Delta A = 1$, the number of subrectangles is 32.

2. $\iint_R f\,dA \approx S_{1,1} = 0.16$ **3.** $\iint_R 5\,dA = 50$

4. The signed volume between the graph $z = f(x, y)$ and the xy-plane. The region below the xy-plane is treated as negative volume.

5. (b) **6.** (b), (c)

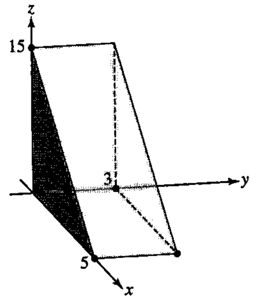
Section 15.1 Exercises

1. $S_{4,3} = 13.5$ **3.** (A) $S_{3,2} = 42$, (B) $S_{3,2} = 43.5$

5. (A) $S_{3,2} = 60$, (B) $S_{3,2} = 62$

7. Two possible solutions are $S_{3,2} = \frac{77}{72}$ and $S_{3,2} = \frac{79}{72}$.

9. $\frac{225}{2}$



11. 0.19375 **13.** 1.0731, 1.0783, 1.0809 **15.** 0 **17.** 0 **19.** 40 **21.** 55 **23.** $\frac{4}{3}$ **25.** 84 **27.** 4 **29.** $\frac{1858}{15}$

31. $6\ln 6 - 2\ln 2 - 5\ln 5 \approx 1.317$ **33.** $\frac{4}{3}\left(19 - 5\sqrt{5}\right) \approx 10.426$

35. $\frac{1}{2}(\ln 3)(-2 + \ln 48) \approx 1.028$ **37.** $6\ln 3 \approx 6.592$

39. 1 **41.** $\left(e^2 - 1\right)\left(1 - \frac{\sqrt{2}}{2}\right) \approx 1.871$ **43.** $m = \frac{3}{4}$

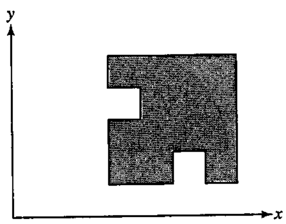
45. $\frac{2}{15}\left(8\sqrt{2} - 7\right) \approx 0.575$ **47.** $2\ln 2 - 1 \approx 0.386$

51. $\frac{e^3}{3} - \frac{1}{3} - e + 1 \approx 4.644$

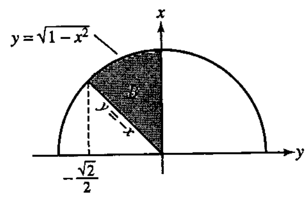
Section 15.2 Preliminary Questions

1. (b), (c)

2.



3.



4. (b)

Section 15.2 Exercises

1. **(a)** Sample points •, $S_{3,4} = -3$
(b) Sample points ∘, $S_{3,4} = -4$

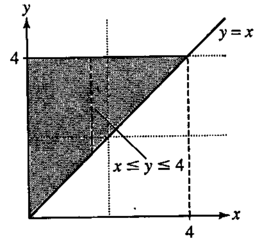
3. As a vertically simple region: $0 \le x \le 1$, $0 \le y \le 1 - x^2$; as a horizontally simple region: $0 \le y \le 1$, $0 \le x \le \sqrt{1-y}$,
$\int_0^1 \left(\int_0^{1-x^2} (xy)\,dy\right) dx = \frac{1}{12}$

5. $\frac{192}{5} = 38.4$ **7.** $\frac{608}{15} \approx 40.53$ **9.** $2\frac{1}{4}$ **11.** $-\frac{3}{4} + \ln 4$

13. $\frac{16}{3} \approx 5.33$ **15.** $\frac{11}{60}$ **17.** $\frac{1754}{15} \approx 116.93$ **19.** $\frac{e-2}{2} \approx 0.359$

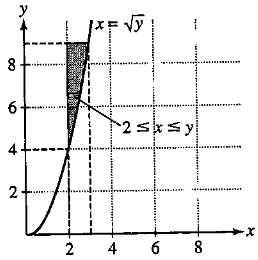
21. $\frac{1}{12}$ **23.** $2e^{12} - \frac{1}{2}e^9 + \frac{1}{2}e^5 \approx 321{,}532.2$

25.



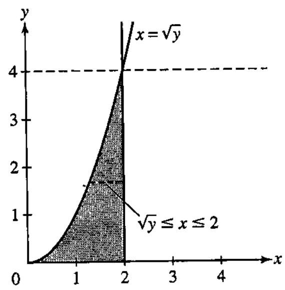
$$\int_0^4 \int_x^4 f(x, y)\,dy\,dx = \int_0^4 \int_0^y f(x, y)\,dx\,dy$$

27.



$$\int_4^9 \int_2^{\sqrt{y}} f(x,\ y)\,dx\,dy = \int_2^3 \int_{x^2}^9 f(x,\ y)\,dy\,dx$$

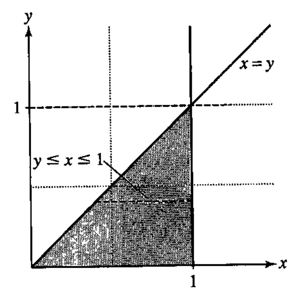
29.



$$\int_0^2 \int_0^{x^2} \sqrt{4x^2+5y}\,dy\,dx = \frac{152}{15}$$

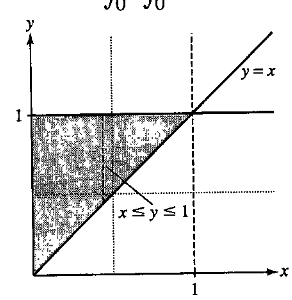
31. $\int_1^e \int_{\ln^2 y}^{\ln y} (\ln y)^{-1}\,dx\,dy = e - 2 \approx 0.718$

33.



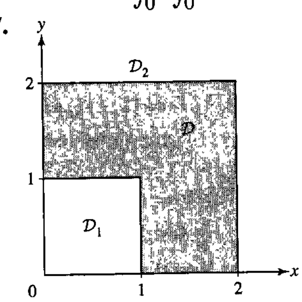
$$\int_0^1 \int_0^x \frac{\sin x}{x}\,dy\,dx = 1 - \cos 1 \approx 0.460$$

35.



$$\int_0^1 \int_0^y x e^{y^3}\,dx\,dy = \frac{e-1}{6} \approx 0.286$$

37.



$$\iint_{\mathcal{D}} e^{x+y}\,dA = e^4 - 3e^2 + 2e \approx 37.878$$

39. $\int_0^4 \int_{x/4}^{3x/4} e^{x^2}\,dy\,dx = \frac{1}{4}\left(e^{16} - 1\right)$

41. $\int_2^4 \int_{y-1}^{7-y} \frac{x}{y^2}\,dx\,dy = 6 - 6\ln 2 \approx 1.841$

43. $\iint_{\mathcal{D}} \frac{\sin y}{y}\,dA = \cos 1 - \cos 2 \approx 0.956$

45. $\int_{-2}^2 \int_0^{4-x^2} (40 - 10y)\,dy\,dx = 256$ **47.** $\frac{512}{3}$

49. $\int_{-2}^2 \left(\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left[\left(8 - x^2 - y^2\right) - \left(x^2 + y^2\right)\right] dy \right) dx$

51. $\int_0^1 \int_0^1 e^{x+y}\,dx\,dy = e^2 - 2e + 1 \approx 2.952$

53. $\frac{1}{\pi} \int_0^1 \int_0^\pi y^2 \sin x\,dx\,dy = \frac{2}{3\pi}$ **55.** $\bar{f} = p$

61. One possible solution is $P = \left(\frac{2}{3},\ 2\right)$.

63. $\iint_{\mathcal{D}} f(x,\ y)\,dA \approx 57.01$

Section 15.3 Preliminary Questions

1. (c) **2.** (b)

3. **(a)** $\mathcal{D} = \{(x,\ y) : 0 \le x \le 1,\ 0 \le y \le x\}$

(b) $\mathcal{D} = \left\{(x,\ y) : 0 \le x \le 1,\ 0 \le y \le \sqrt{1-x^2}\right\}$

Section 15.3 Exercises

1. 6 **3.** $(e-1)(1-e^{-2})$ **5.** $-\frac{27}{4} = -6.75$

7. $\frac{b}{20}\left[(a+c)^5 - a^5 - c^5\right]$ **9.** $\frac{1}{6}$ **11.** $\frac{1}{16}$ **13.** $e - \frac{5}{2}$

15. $2\frac{1}{12}$ **17.** $\frac{128}{15}$ **19.** $\int_0^3 \int_0^4 \int_0^{y/4} dz\,dy\,dx = 6$

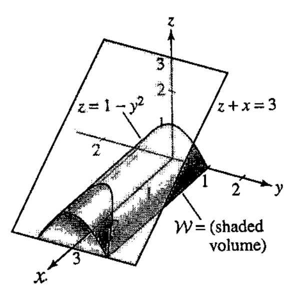
21. $\frac{1}{12}$ **23.** $\frac{126}{5}$

25. Region enclosed by sphere $x^2 + y^2 + z^2 = 5$ to right of plane $y = 1$.

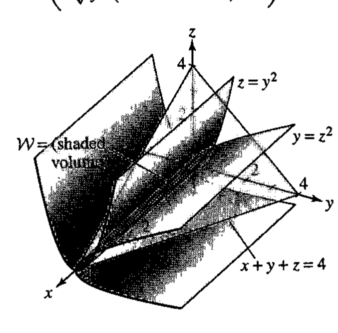
27. $\int_0^2 \int_0^{y/2} \int_0^{4-y^2} xyz\,dz\,dx\,dy$, $\int_0^4 \int_0^{\sqrt{4-z}} \int_0^{y/2} xyz\,dx\,dy\,dz$, and $\int_0^4 \int_0^{\sqrt{1-(z/4)}} \int_{2x}^{\sqrt{4-z}} xyz\,dy\,dx\,dz$

29. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 f(x,\ y,\ z)\,dz\,dy\,dx$ **31.** $\frac{16}{21}$

33. $\int_{-1}^1 \left(\int_0^{1-y^2} \left(\int_0^{3-z} dx \right) dz \right) dy$



35. $\int_0^1 \left(\int_{\sqrt{y}}^{y^2} \left(\int_0^{4-y-z} dx \right) dz \right) dy$



. $\frac{1}{2\pi}$ **39.** $2e - 4 \approx 1.437$

41. $S_{N,N,N} \approx 0.561,\ 0.572,\ 0.576;\ I \approx 0.584;\ N = 100$

Section 15.4 Preliminary Questions

1. (d)

2. (a) $\int_{-1}^{2} \int_0^{2\pi} \int_0^2 f(P)\, r\, dr\, d\theta\, dz$

(b) $\int_{-2}^{0} \int_0^{2\pi} \int_0^{\sqrt{4-z^2}} r\, dr\, d\theta\, dz$

3. (a) $\int_0^{2\pi} \int_0^{\pi} \int_0^4 f(P)\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

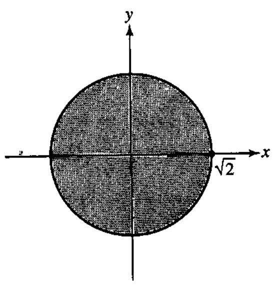
(b) $\int_0^{2\pi} \int_0^{\pi} \int_4^5 f(P)\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

(c) $\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_0^2 f(P)\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

4. $\Delta A \approx r(\Delta r \Delta\theta)$, and the factor r appears in $dA = r\, dr\, d\theta$ in the Change of Variables formula.

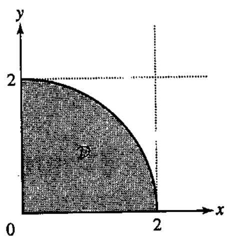
Section 15.4 Exercises

1.



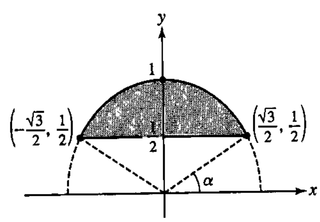
$$\iint_D \sqrt{x^2 + y^2}\, dA = \frac{4\sqrt{2}\pi}{3}$$

3.



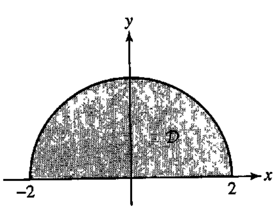
$$\iint_D xy\, dA = 2$$

5.



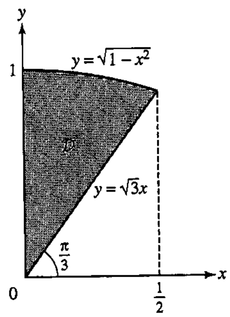
$$\iint_D y\left(x^2 + y^2\right)^{-1} dA = \sqrt{3} - \frac{\pi}{3} \approx 0.685$$

7.



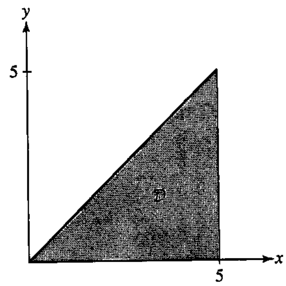
$$\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} (x^2 + y^2)\, dy\, dx = 4\pi$$

9.



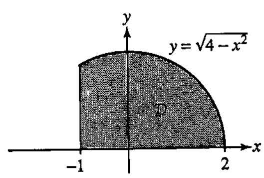
$$\int_0^{1/2} \int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\, dy\, dx = \tfrac{1}{3}\left(1 - \tfrac{\sqrt{3}}{2}\right) \approx 0.045$$

11.



$$\int_0^{\pi/4} \left(\int_0^{5/\cos(\theta)} \left(r^2 \cos(\theta)\right) dr\right) d\theta = \frac{125}{3}$$

13.



$$\int_{-1}^{2} \int_0^{\sqrt{4-x^2}} (x^2 + y^2)\, dy\, dx = \tfrac{\sqrt{3}}{2} + \tfrac{8\pi}{3} \approx 9.244$$

15. $\frac{1}{4}$ **17.** $\frac{1}{2}$ **19.** 0 **21.** 18 **23.** $\frac{48\pi - 32}{9} \approx 13.2$

25. (a) $W: 0 \le \theta \le 2\pi,\ 0 \le r \le 2,\ 2 \le z \le 6 - r^2$ (b) 8π

27. $\frac{405\pi}{2} \approx 636.17$ **29.** $\frac{2}{3}$ **31.** 243π

33. $\int_0^{2\pi} \int_0^1 \int_0^4 f(r\cos\theta,\ r\sin\theta,\ z)\, r\, dz\, dr\, d\theta$

35. $\int_0^{\pi} \int_0^1 \int_0^{r^2} f(r\cos\theta,\ r\sin\theta,\ z)\, r\, dz\, dr\, d\theta$

37. $z = \frac{H}{R} r;\ V = \frac{\pi R^2 H}{3}$ **39.** 16π

41. $V =$
$2\int_0^{2\pi} \left(\int_b^a \left(\int_0^{\sqrt{a^2-r^2}} (r)\, dz\right) dr\right) d\theta = \frac{4}{3}\pi\left(a^2 - b^2\right)^{3/2} = \frac{4}{3}\pi r h^3$

43. $V = \int_0^{2\pi} \left(\int_0^{\pi/3} \left(\int_{\sec(\phi)}^{2} \left(\rho^2 \sin(\phi)\right) d\rho\right) d\phi\right) d\theta = \frac{5\pi}{3}$

45. $-\frac{\pi}{16}$ **47.** $\frac{8\pi}{15}$ **49.** $\frac{8\pi}{5}$ **51.** $\frac{5\pi}{8}$ **53.** π **55.** $\frac{4\pi a^3}{3}$

57. (b) $J = \int_0^{2\pi} \left(\int_0^{\infty} \left(e^{-r^2} r\right) dr\right) d\theta = \pi$

Section 15.5 Preliminary Questions

1. 5 kg/m^3 **2.** (a)

3. The probability that $0 \le X \le 1$ and $0 \le Y \le 1$; the probability that $0 \le X + Y \le 1$

Section 15.5 Exercises

1. $\frac{2}{3}$ **3.** $4\left(1 - e^{-100}\right) \times 10^{-6}$ C $\approx 4 \times 10^{-6}$ C

5. $10{,}000 - 18{,}000e^{-4/5} \approx 1{,}912$

7. $25\pi\left(3 \times 10^{-8}\text{ C}\right) \approx 2.356 \times 10^{-6}$ C

9. $\approx 2.593 \times 10^{10}$ kg **11.** $\left(0, \frac{2}{5}\right)$ **13.** $\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$

15. $(0.555, 0)$ **17.** $\left(0, 0, \frac{3R}{8}\right)$ **19.** $\left(0, 0, \frac{9}{8}\right)$

21. $\left(0, 0, \frac{13}{2\left(17-6\sqrt{6}\right)}\right)$ **23.** $(2, 1)$ **25.** $\left(\frac{1}{6}, \frac{1}{6}\right)$ **27.** $\frac{16}{15\pi}$

29. **(a)** $\frac{M}{4ab}$ **(b)** $I_x = \frac{Mb^2}{3}$; $I_0 = \frac{M(a^2+b^2)}{3}$ **(c)** $\frac{b}{\sqrt{3}}$

31. $I_0 = 8{,}000$ kg $\cdot$ m^2; $I_x = 4{,}000$ kg $\cdot$ m^2

33. $\frac{9}{2}$ **35.** $\frac{243}{20}$ **37.** $\left(\frac{a}{2}, \frac{2b}{5}\right)$ **39.** $\frac{a^2b^4}{60}$

41. $I_x = \frac{MR^2}{4}$; kinetic energy required is $\frac{25MR^2}{2}$ J

47. **(a)** $I = 182.5$ g $\cdot$ cm^2 **(b)** $\omega \approx 126.92$ rad/s

49. $\frac{13}{72}$ **51.** $\frac{1}{64}$ **53.** $C = 15$; probability is $\frac{5}{8}$.

55. **(a)** $C = 4$ **(b)** $\frac{1}{48\pi} + \frac{1}{32} \approx 0.038$

Section 15.6 Preliminary Questions

1. (b)

2. **(a)** $G(1, 0) = (2, 0)$ **(b)** $G(1, 1) = (1, 3)$
(c) $G(2, 1) = (3, 3)$

3. Area $(G(R)) = 36$ **4.** Area $(G(R)) = 0.06$

Section 15.6 Exercises

1. **(a)** Image of the u-axis is the line $y = \frac{1}{2}x$; image of the v-axis is the y-axis.

(b) The parallelogram with vertices $(0, 0)$, $(10, 5)$ $(10, 2)$, $(0, 7)$

(c) The segment joining the points $(2, 3)$ and $(10, 8)$

(d) The triangle with vertices $(0, 1)$, $(2, 1)$, and $(2, 2)$

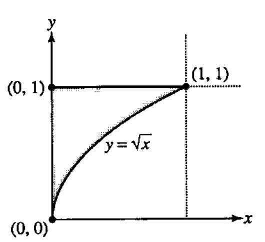
3. G is not one-to-one; G is one-to-one on the domain $\{(u, v) : u \geq 0\}$, and G is one-to-one on the domain $\{(u, v) : u \leq 0\}$.

(a) The positive x-axis including the origin and the y-axis, respectively

(b) The rectangle $[0, 1] \times [-1, 1]$

(c) The curve $y = \sqrt{x}$ for $0 \leq x \leq 1$

(d)



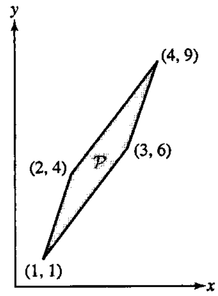
5. $y = 3x - c$ **7.** $y = \frac{17}{6}x$ **11.** Jac$(G) = 1$
13. Jac$(G) = -10$ **15.** Jac$(G) = 1$ **17.** Jac$(G) = 4$

19. $G(u, v) = (4u + 2v, u + 3v)$ **21.** $\frac{2329}{12} \approx 194.08$

23. **(a)** Area $(G(R)) = 105$ **(b)** Area $(G(R)) = 126$

25. Jac$(G) = \frac{2u}{v}$; for $R = [1, 4] \times [1, 4]$, area $(G(R)) = 15 \ln 4$

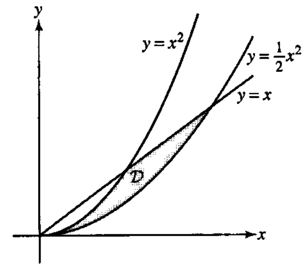
27.



$G(u, v) = (1 + 2u + v, 1 + 5u + 3v)$

29. 82 **31.** 80 **33.** $\frac{56}{45}$ **35.** $\frac{\pi(e^{36} - 1)}{6}$

37.



$$\iint_D y^{-1}\, dx\, dy = 1$$

39. $\iint_D e^{xy}\, dA = (e^{20} - e^{10}) \ln 2$

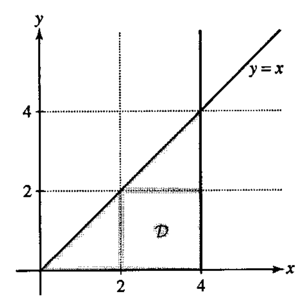
41. **(b)** $-\frac{1}{x+y}$ **(c)** $I = 9$ **45.** $\frac{\pi^2}{8}$

Chapter 15 Review

1. **(a)** $S_{2,3} = 240$ **(b)** $S_{2,3} = 510$ **(c)** 520

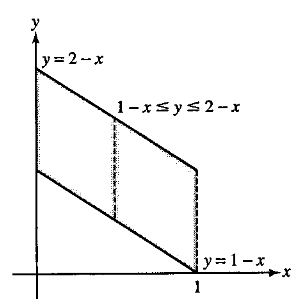
3. $S_{4,4} = 2.9375$ **5.** $\frac{32}{3}$ **7.** $\frac{\sqrt{3} - 1}{2}$

9.



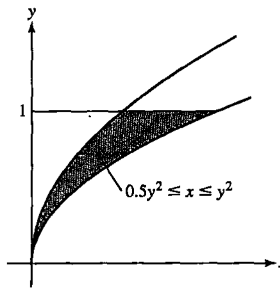
$$\iint_D \cos y\, dA = 1 - \cos 4$$

11.



$$\iint_D e^{x+2y}\, dA = \frac{1}{2}e(e+1)(e-1)^2$$

13.



$$\iint_D ye^{1+x}\,dA = 0.5(e^2 - 2e^{1.5} + e)$$

15. $\int_0^9 \int_{-\sqrt{9-y}}^{\sqrt{9-y}} f(x, y)\,dx\,dy$ **17.** $\frac{1}{24}$ **19.** $18(\sqrt{2}-1)$

21. $1-\cos 1$ **23.** 6π **25.** $\pi/2$ **27.** 10 **29.** $\frac{\pi}{4}+\frac{2}{3}$ **31.** π

33. $\frac{1}{4}$ **35.** $\int_0^{\pi/2}\int_0^1\int_0^r zr\,dz\,dr\,d\theta = \pi/16$ **37.** $\frac{2\pi(-1+e^8)}{3e^8}$

41. $\frac{256\pi}{15} \approx 53.62$ **43.** 1280π **45.** $\left(-\frac{1}{4}R,\ 0,\ \frac{5}{8}H\right)$

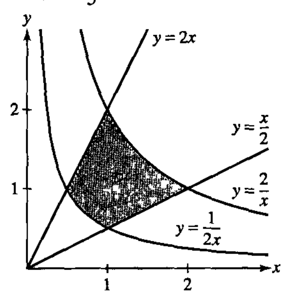
47. $\left(-\frac{2}{11\pi}R,\ -\frac{2}{11\pi}R(2-\sqrt{3}),\ \frac{1}{2}H\right)$. **49.** $\left(0,\ 0,\ \frac{2}{3}\right)$

51. $\left(\frac{8}{15},\ \frac{16}{15\pi},\ \frac{16}{15\pi}\right)$ **53.** $\frac{19}{33}$ **55.** $\frac{4}{7}$

57. $G(u, v) = (3u+v,\ -u+4v)$; Area $(G(R)) = 156$

59. Area$(D) \approx \frac{1}{5}$

61. (a)

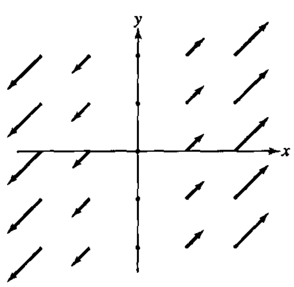


(d) $\frac{3}{4}(e^2 - \sqrt{e})$

Chapter 16

Section 16.1 Preliminary Questions

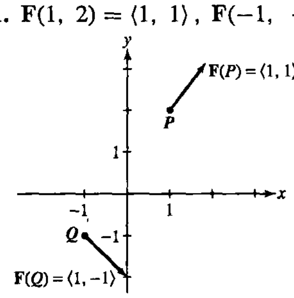
1. (b) **2.**



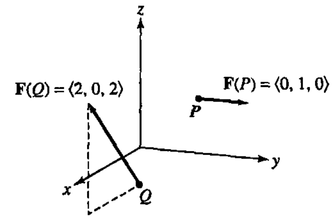
3. $\mathbf{F} = \langle 0,\ -z,\ y\rangle$ **4.** $f_1(x, y, z) = xyz + 1$

Section 16.1 Exercises

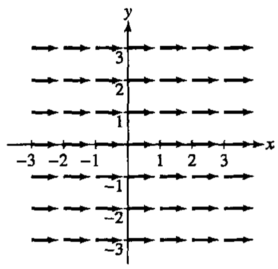
1. $\mathbf{F}(1, 2) = \langle 1, 1\rangle$, $\mathbf{F}(-1, -1) = \langle 1, -1\rangle$



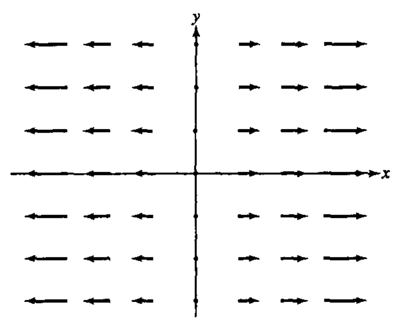
3. $\mathbf{F}(P) = \langle 0, 1, 0\rangle$, $\mathbf{F}(Q) = \langle 2, 0, 2\rangle$



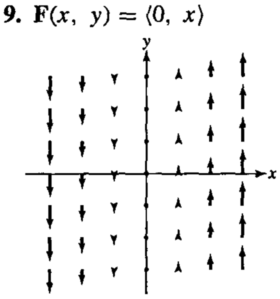
5. $\mathbf{F} = \langle 1, 0\rangle$



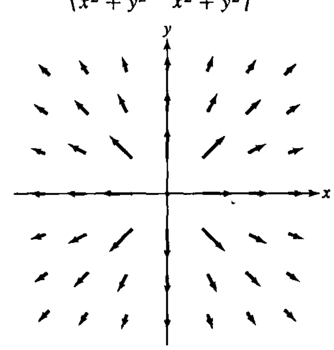
7. $\mathbf{F} = x\mathbf{i}$



9. $\mathbf{F}(x, y) = \langle 0, x\rangle$



11. $\mathbf{F} = \left\langle \frac{x}{x^2+y^2},\ \frac{y}{x^2+y^2}\right\rangle$



13. Plot (D) **15.** Plot (B) **17.** Plot (C) **19.** Plot (B)

21. $(0, y)$ **23.** $\text{div}(\mathbf{F}) = y + z$, $\text{curl}(\mathbf{F}) = \left\langle y, 3x^2, -x\right\rangle$

25. $\text{div}(\mathbf{F}) = 1 - 4xz - x + 2x^2z$, $\text{curl}(\mathbf{F}) = \left\langle -1, 2x^2 - 2xz^2, -y\right\rangle$

27. $\text{div}(\mathbf{F}) = 0$, $\text{curl}(\mathbf{F}) = \left\langle 1 - 3z^2, 1 - 2x, 1 + 2y\right\rangle$

29. $\text{div}(\mathbf{F}) = 0$, $\text{curl}(\mathbf{F}) = \left\langle 0, \sin x, \cos x - e^y\right\rangle$

39. $f(x, y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + K$
41. $f(x, y) = e^{xy} + K$
43. $f(x, y, z) = xyz^2 + K$ **45.** $f(x, y, z) = \sin(xyz) + K$
47. $f(x, y, z) = ax + by + cz + K$
49. **(b)** $e_P(1, 1) = \langle -2, -1 \rangle / \sqrt{5}$;
(c) $f(x, y, z) = \sqrt{(x - a)^2 + (y - b)^2}$
51. (A)

Section 16.2 Preliminary Questions

1. 50 **2.** (a), (c), (d), (e)
3. **(a)** True
(b) False. Reversing the orientation of the curve changes the sign of the vector line integral.
4. **(a)** 0 **(b)** −5

Section 16.2 Exercises

1. **(a)** $f(\mathbf{r}(t)) = 6t + 4t^2$; $ds = 2\sqrt{11}\,dt$
(b) $\int_0^1 \left(6t + 4t^2\right) 2\sqrt{11}\,dt = \frac{26\sqrt{11}}{3}$
3. **(a)** $\mathbf{F}(\mathbf{r}(t)) = \left\langle t^{-2}, t^2 \right\rangle$; $d\mathbf{r} = \left\langle 1, -t^{-2} \right\rangle dt$
(b) $\int_1^2 \left(t^{-1} - 1\right) dt = -\frac{1}{2}$
5. $\sqrt{2}\left(\pi + \frac{\pi^3}{3}\right)$ **7.** $\pi^2/2$ **9.** 2.8 **11.** $\frac{128\sqrt{29}}{3} \approx 229.8$
13. $\frac{\sqrt{3}}{2}(e - 1) \approx 1.488$ **15.** $\frac{2}{3}\left((e^2 + 5)^{3/2} - 2^{3/2}\right)$
17. 39; the distance between (8, −6, 24) and (20, −15, 60)
19. $\frac{16}{3}$ **21.** 0 **23.** $2(e^2 - e^{-2}) - (e - e^{-1}) \approx 12.157$ **25.** $\frac{10}{9}$
27. $-\frac{8}{3}$ **29.** $\frac{13}{2}$ **31.** $\frac{\pi}{2}$ **33.** 339.5587 **35.** $2 - e - \frac{1}{e}$
37. **(a)** −8 **(b)** −11 **(c)** −16
39. ≈7.6; ≈4 **41.** (A) Zero, (B) Negative, (C) Zero **43.** 64π g
45. $\approx 10.4 \times 10^{-6}$ C **47.** ≈22,743.10 volts **49.** ≈−10,097 volts
51. 1 **53.** $\frac{27}{28}$ **55.** **(a)** *ABC* **(b)** *CBA*
59. $\frac{1}{3}\left((4\pi^2 + 1)^{3/2} - 1\right) \approx 85.5 \times 10^{-6}$ C **65.** 18 **67.** $e - 1$

Section 16.3 Preliminary Questions

1. Closed
2. **(a)** Conservative vector fields **(b)** All vector fields
(c) Conservative vector fields **(d)** All vector fields
(e) Conservative vector fields **(f)** All vector fields
(g) Conservative vector fields and *some* other vector fields
3. **(a)** Always true **(b)** Always true
(c) True under additional hypotheses on D
4. **(a)** 4 **(b)** −4

Section 16.3 Exercises

1. 0 **3.** $-\frac{9}{4}$ **5.** $32e - 1$ **7.** $f(x, y, z) = zx + y$
9. $f(x, y, z) = y^2x + e^zy$
11. The vector field is not conservative.
13. $f(x, y, z) = z\tan x + zy$ **15.** $f(x, y, z) = x^2y + 5x - 4zy$
17. 16 **19.** 1 **21.** 6 **23.** $\frac{2}{3}$; 0 **25.** 6.2×10^9 J
27. **(a)** $f(x, y, z) = -gz$ **(b)** ≈ 82.8 m/s
29. (A) 2π, (B) 2π, (C) 0, (D) -2π, (E) 4π

Section 16.4 Preliminary Questions

1. 50
2. A distortion factor that indicates how much the area of R_{ij} is altered under the map G.
3. Area$(\mathcal{S}) \approx 0.0006$ **4.** $\iint_{\mathcal{S}} f(x, y, z)\,dS \approx 0.6$
5. Area$(\mathcal{S}) = 20$ **6.** $\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$

Section 16.4 Exercises

1. **(a)** v **(b)** iii **(c)** i **(d)** iv **(e)** ii
3. **(a)** $\mathbf{T}_u = \langle 2, 1, 3 \rangle$, $\mathbf{T}_v = \langle 0, -1, 1 \rangle$, $\mathbf{n}(u, v) = \langle 4, -2, -2 \rangle$
(b) Area$(\mathcal{S}) = 4\sqrt{6}$ **(c)** $\iint_{\mathcal{S}} f(x, y, z)\,dS = \frac{32\sqrt{6}}{3}$
5. **(a)** $\mathbf{T}_x = \langle 1, 0, y \rangle$, $\mathbf{T}_y = \langle 0, 1, x \rangle$, $\mathbf{N}(x, y) = \langle -y, -x, 1 \rangle$
(b) $\frac{(2\sqrt{2} - 1)\pi}{6}$ **(c)** $\frac{\sqrt{2} + 1}{15}$
7. $\mathbf{T}_u = \langle 2, 1, 3 \rangle$, $\mathbf{T}_v = \langle 1, -4, 0 \rangle$, $\mathbf{N}(u, v) = 3\langle 4, 1, -3 \rangle$, $4x + y - 3z = 0$
9. $\mathbf{T}_\theta = \langle -\sin\theta\sin\phi, \cos\theta\sin\phi, 0 \rangle$,
$\mathbf{T}_\phi = \langle \cos\theta\cos\phi, \sin\theta\cos\phi, -\sin\phi \rangle$,
$\mathbf{N}(u, v) = -\cos\theta\sin^2\phi\mathbf{i} - \sin\theta\sin^2\phi\mathbf{j} - \sin\phi\cos\phi\mathbf{k}$,
$y + z = \sqrt{2}$
11. Area$(\mathcal{S}) \approx 0.2078$ **13.** $\frac{\sqrt{2}}{5}$ **15.** $\frac{37\sqrt{37} - 1}{4} \approx 56.02$ **17.** $\frac{\pi}{6}$
19. $4\pi(1 - e^{-4})$ **21.** $\frac{\sqrt{3}}{6}$ **23.** $\frac{7\pi}{3}$ **25.** $\frac{5\sqrt{10}}{27} - \frac{1}{54}$
27. Area$(\mathcal{S}) = 16$ **29.** $3e^3 - 6e^2 + 3e + 1 \approx 25.08$
31. Area$(\mathcal{S}) = 4\pi R^2$
33. **(a)** Area$(\mathcal{S}) \approx 1.0780$ **(b)** ≈ 0.09814
35. Area$(\mathcal{S}) = \frac{5\sqrt{29}}{4} \approx 6.73$ **37.** Area$(\mathcal{S}) = \pi$ **39.** 48π
43. Area$(\mathcal{S}) = \frac{\pi}{6}\left(17\sqrt{17} - 1\right) \approx 36.18$ **47.** $4\pi^2ab$
49. $f(r) = -\frac{Gm}{2Rr}\left(\sqrt{R^2 + r^2} - |R - r|\right)$

Section 16.5 Preliminary Questions

1. (b) **2.** (c) **3.** (a) **4.** (b)
5. **(a)** 0 **(b)** π **(c)** π
6. $\approx 0.05\sqrt{2} \approx 0.0707$ **7.** 0

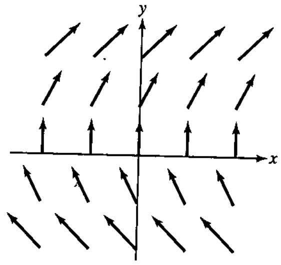
Section 16.5 Exercises

1. **(a)** $\mathbf{N} = \langle 2v, -4uv, 1 \rangle$, $\mathbf{F} \cdot \mathbf{N} = 2v^3 + u$
(b) $\frac{4}{\sqrt{69}}$ **(c)** 265
3. 4 **5.** −4 **7.** $\frac{27}{12}(3\pi + 4)$ **9.** $\frac{693}{5}$ **11.** $\frac{11}{12}$ **13.** $\frac{9\pi}{4}$
15. $(e - 1)^2$ **17.** 270
19. **(a)** $18\pi e^{-3}$ **(b)** $\frac{\pi}{2}e^{-1}$
21. $\left(2 - \frac{6}{\sqrt{13}}\right)\pi k$ **23.** $\frac{2\pi}{3}$ m³/s **25.** 4π **27.** $\frac{16\pi}{3}$
29. **(a)** 1 **(b)** 1
33. $\Phi(t) = -1.56 \times 10^{-5}e^{-0.1t}$ T-m²;
voltage drop $= -1.56 \times 10^{-6}e^{-0.1t}$ V
35. The flow is greatest at $z = 3$.

Chapter 16 Review

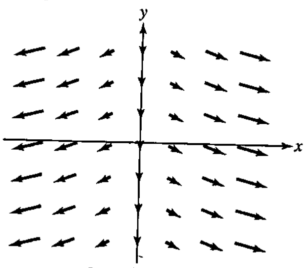
1. **(a)** $\langle -15, 8\rangle$ **(b)** $\langle 4, 8\rangle$ **(c)** $\langle 9, 1\rangle$

3.



$\mathbf{F} = \langle y, 1\rangle$

5. $\mathbf{F}(x, y) = \langle 2x, -1\rangle$



$\nabla V = \langle 2x, -1\rangle$

7. $\text{div}(\mathbf{F}) = 2x + 2y + 2z$, $\text{curl}(\mathbf{F}) = \langle 0, 0, 0\rangle$

9. $\text{div}(\mathbf{F}) = 3x^2y + y^2$, $\text{curl}(\mathbf{F}) = \left\langle 2yz - 2xz, 0, z^2 - x^3\right\rangle$

11. $\text{div}(\mathbf{F}) = -1$, $\text{curl}(\mathbf{F}) = \langle 0, 0, -1\rangle$

13. $\text{div}(\mathbf{F}) = 4x^2e^{-x^2-y^2-z^2} + 4y^2e^{-x^2-y^2-z^2} + 4z^2e^{-x^2-y^2-z^2} - 6e^{-x^2-y^2-z^2}$; $\text{curl}(\mathbf{F}) = \langle 0, 0, 0\rangle$

15. $\text{curl}(\mathbf{F}) = \langle -2z, 0, 2y\rangle$ **19.** $f(x, y) = x^4y^5$

21. $\mathbf{F}$ is conservative, $f(x, y, z) = 2x + 4y + e^z$

23. $f(x, y) = \frac{x^4y^4}{4}$

25. $\mathbf{F}$ is conservative, $f(x, y, z) = y\ln\left(x^2 + z\right)$

27. $\mathbf{F} = \langle 1 + by, 1 + bx\rangle$ **29.** -2

31. $\sqrt{2}(\sin 3 - 2\cos 3 + \sin 1 + 2\cos 1 + 4) \approx 11.375$

33. **(a)** 2 **(b)** 0 **37.** $\frac{1}{3}$ **39.** $4\ln\left(1 + (\ln 2)^4 + e^2\right) \approx 8.616$

41. $\frac{52}{29} \approx 1.79$ **43.** $3e^{3/2} - \frac{15}{2} \approx 5.945$

45. Area $(\mathcal{S}) =$

$\int_{-1}^{1}\int_{-1}^{1}\sqrt{125u^2 - 100uv + 425v^2 + 81}\, du\, dv \approx 62.911$

47. $\frac{4}{9}\left(2\sqrt{2} - 1\right)$

49. **(a)** Zero since $f(x, y, z) = y^3$ is odd and symmetric about the xz-plane

(b) Positive since $f(x, y, z) = z^3$ is non-negative

(c) Zero since $f(-x, y, z) = -xyz = -f(x, y, z)$ is symmetric about the yz-plane

(d) Negative since $f(x, y, z) = z^2 - 2$ is negative

51. **(a)** $\mathbf{N} = \left\langle \frac{2}{\sqrt{5}}, \frac{2\sqrt{3}}{\sqrt{5}}, 2\right\rangle$

(b) Area $(G(R)) = \frac{6}{\sqrt{5}} \cdot 0.1 \cdot 0.05 \approx 0.0134$

53. $\int\int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} =$

$\int_0^1\left(\int_0^1 (-4(2u - v) - 2(2v + 5) + 7(-u - 3v))\, dv\right) du = -28$

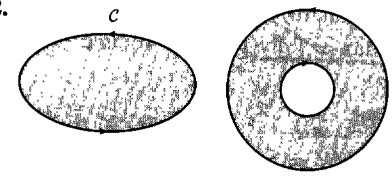
55. $\frac{1}{4} - \frac{\sinh 1}{3}$ **57.** $\frac{\sqrt{6}\pi}{9}$ **59.** 27π **61.** $\frac{9\sqrt{3}}{4}$

Chapter 17

Section 17.1 Preliminary Questions

1. $\mathbf{F} = \left\langle -e^y, x^2\right\rangle$

2.



3. Yes **4.** (a), (c)

Section 17.1 Exercises

1. $\oint_{\mathcal{C}} xy\, dx + y\, dy =$

$\int_0^{2\pi}(\cos\theta\sin\theta(-\sin\theta) + \sin\theta\cos\theta)\, d\theta = 0 =$

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}(0 - x)\, dy\, dx = \int\int_{\mathcal{D}}\left(\frac{\partial}{\partial x}y - \frac{\partial}{\partial y}(xy)\right)dA$

3. 0 **5.** $-\frac{\pi}{4}$ **7.** $\frac{1}{6}$ **9.** $\frac{(e^2 - 1)(e^4 - 5)}{2}$

11. (a) $V(x, y) = x^2e^y$ **13.** $I = 34$ **15.** $\frac{1}{2}$ **17.** $\frac{8}{3}$

19. (c) $A = \frac{3}{2}$ **23.** $9 + \frac{15\pi}{2}$ **25.** 214π

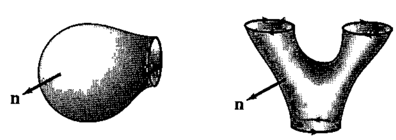
27. (A) Zero (B) Positive (C) Negative (D) Zero

29. -0.10 **31.** $R = \sqrt{\frac{2}{3}}$ **33.** Triangle (A), 3; Polygon (B), 12

35. -4 **37.** 5π **39.** $\frac{9\pi}{4}$ **41.** 29.2 buffalo per minute

Section 17.2 Preliminary Questions

1.



(A) (B)

2. (a)

3. A vector field $\mathbf{A}$ such that $\mathbf{F} = \text{curl}(\mathbf{A})$ is a vector potential for $\mathbf{F}$

4. (b)

5. $\mathbf{F}$ must be the curl of some other vector field $\mathbf{A}$ (see Theorem 2).

Section 17.2 Exercises

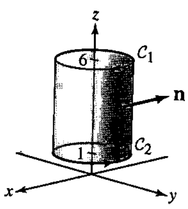
1. $\int\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s} = \int\int_{\mathcal{S}} \text{curl}(\mathbf{F}) \cdot d\mathbf{S} = \pi$

3. $\int\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s} = \int\int_{\mathcal{S}} \text{curl}(\mathbf{F}) \cdot d\mathbf{S} = e^{-1} - 1$

5. $\left\langle -3z^2e^{z^3}, 2ze^{z^2} + z\sin(xz), 2\right\rangle$; 2π

7. $\langle -2, 3, 5\rangle$, 20π **9.** 0 **11.** -45π **13.** 0

15. **(a)**



(b) 140π

17. **(a)** $\mathbf{A} = \left\langle 0, 0, e^y - e^{x^2}\right\rangle$ **(c)** $\int\int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = \frac{\pi}{2}$

19. **(a)** $\int\int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{S} = r^2B\pi$ **(b)** $\int_{\partial\mathcal{S}} \mathbf{A} \cdot d\mathbf{s} = 0$

21. $\int\int_{\mathcal{S}} B\, dS = 18b$ **23.** $c = 2a$ and b is arbitrary.

27. $\int\int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = 25$

Section 17.3 Preliminary Questions

1. $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$

2. Since the integrand is positive for all $(x, y, z) \neq (0, 0, 0)$, the triple integral, hence also the flux, is positive.

3. (a), (b), (d), (f) are meaningful; (b) and (d) are automatically zero.

4. (c) **5.** $\text{div}(\mathbf{F}) = 1$ and flux $= \int \text{div}(\mathbf{F})\, dV =$ volume

Section 17.3 Exercises

1. $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_{\mathcal{R}} \text{div}(\mathbf{F})\, dV = \iiint_{\mathcal{R}} 0\, dV = 0$

3. $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_{\mathcal{R}} \text{div}(\mathbf{F})\, dV = 4\pi$ **5.** $\frac{4\pi}{15}$ **7.** 60π

9. $\frac{3{,}616}{105}$ **11.** $\frac{32\pi}{5}$ **13.** 64π **15.** 81π **17.** π **19.** $\frac{13}{3}$

21. $\frac{4\pi}{3}$ **23.** $\frac{16\pi}{3} + \frac{9\sqrt{3}}{2} \approx 24.549$ **25.** $\approx 1.57\ \text{m}^3/\text{s}$

27. **(b)** 0 **(c)** 0

(d) Since $\mathbf{E}$ is not defined at the origin, which is inside the ball W, we cannot use the Divergence Theorem.

29. $(-4) \cdot \left[\frac{256\pi}{3} - 1\right] \approx -1068.33$

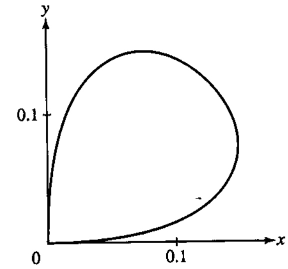
31. $\text{Div}(f\mathbf{F}) = f\text{Div}(\mathbf{F}) + \mathbf{F} \cdot \nabla f$

35. **(d)** Flux through a closed surface is 0.

Chapter 17 Review

1. 0 **3.** -30 **5.** $\frac{3}{5}$

7. **(a)**



(b) $A = \frac{1}{60}$

9. $\frac{1}{3}$ **11.** $\frac{1}{3}$ **13.** $\{(x, y) \mid y = x \text{ or } y = -x\}$ **15.** 36π **19.** 2π

21. $\mathbf{A} = \langle yz, 0, 0\rangle$ and the flux is 8π. **23.** $\frac{296}{3}$ **25.** -128π

27. $\text{Volume}(W) = 2$ **29.** $4 \cdot 0.0009\pi \approx 0.0113$

31. $2x - y + 4z = 0$ **33.** **(b)** $\frac{\pi}{2}$

35. **(c)** 0 **(d)** $\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = -4$, $\int_{C_2} \mathbf{F} \cdot d\mathbf{s} = 4$

41. $V = \frac{4\pi}{3}abc$