Solutions to the “QUIZ” for Sept. 10, 2009

1. Find an equation of the plane that passes through the points \((0, 1, 1), (1, 0, 1), (1, 1, 0)\).

**Sol.** Let’s call \(P = (0, 1, 1), \ Q = (1, 0, 1), \ R = (1, 1, 0)\).

We need **two** direction vectors that lie on that plane. For example

\[
PQ = \langle 1 - 0, 0 - 1, 1 - 1 \rangle = \langle 1, -1, 0 \rangle
\]

\[
PR = \langle 1 - 1, 0 - 1, 0 - 0 \rangle = \langle 1, 0, -1 \rangle
\]

To get the **normal** we take the cross-product \(PQ \times PR\).

\[
PQ \times PR = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix}
\]

\[
= \begin{vmatrix} i & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} - \begin{vmatrix} i & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix} + \begin{vmatrix} i & 1 & -1 \\ 1 & 0 & 0 \end{vmatrix}
\]

\[
= i((-1) \cdot (-1) - 0 \cdot 0) - j(1 \cdot (-1) - 0 \cdot 1) + k(1 \cdot 0 - (-1) \cdot 1)
\]

\[
= i + j + k
\]

Finally converting to the **usual** notation, we get \(n = \langle 1, 1, 1 \rangle\). The equation of a general plane is

\[
a(x - x_0) + b(y - y_0) + c(z - z_0) = 0,
\]

where \(\langle a, b, c \rangle = \langle 1, 1, 1 \rangle\) is the normal vector we just found, and \((x_0, y_0, z_0)\) is **any** point (either \(P, Q\) or \(R\)). Picking \(P\) we get

\[
1(x - 0) + 1(y - 1) + 1(z - 1) = 0
\]

that simplifies to

\[
x + y + z = 2.
\]

**Ans.:** \(x + y + z = 2\).

**Comments:** About %70 got it perfectly. About %10 were clueless. The rest made some calculational mistakes. Quite a few people got \(n = \langle 1, 1, 1 \rangle\) correctly, but messed up with the last part of plugging-in \((x_0, y_0, z_0) = (0, 1, 1),\) and got either \(x + y + z = 0,\) or \(x + y + z = 3\) or other wrong things. Remember that you can always **check** your answer by plugging-in the three points and see that they lie on the plane. For \(P = (0, 1, 1): \ 0 + 1 + 1 = 2,\) for \(Q = (1, 0, 1), \ 1 + 0 + 1 = 2,\) for \(R = (1, 1, 0): \ 1 + 1 + 0 = 2,\) they all agree. People who got, for example \(x + y + z = 3\) could have realized their mistake by doing this checking.
2. Find the intersection of the line

\[ \mathbf{r}(t) = \langle 1, 1, 0 \rangle + t\langle 0, 2, 4 \rangle \]

and the plane

\[ x + y + z = 14 \]

**Solution:** First spell-out \( \mathbf{r}(t) \):

\[ \mathbf{r}(t) = \langle 1, 1, 0 \rangle + t\langle 0, 2, 4 \rangle = \langle 1 + 2t, 1, 4t \rangle , \]

and in scalar form

\[ x = 1 , \quad y = 1 + 2t , \quad z = 4t . \]

Now plug these expressions for \( x, y, z \) in terms of the parameter \( t \) into the equation of the plane \( x + y + z = 14 \) getting

\[ 1 + (1 + 2t) + 4t = 14 \]

Simplifying, we get

\[ 2 + 6t = 14 \]

so

\[ 6t = 12 \]

that gives \( t = 2 \). Having found the lucky \( t \) (namely 2) you plug it in back into

\[ x = 1 , \quad y = 1 + 2\times 2 = 5 , \quad z = 4\times 2 = 8 \]

getting

\[ x = 1 , \quad y = 1 + 2 \cdot 2 = 5 , \quad z = 4 \cdot 2 = 8 \]

So the **lucky point** that belongs both to the plane and the line is the point \( (1, 5, 8) \).

**Ans.:** The intersection of the line and the plane given by the problem is the point \( (1, 5, 8) \).

**Comments:** About \( 85\% \) got it perfectly. A few people forgot to plug-in \( t = 2 \) back into \( x, y, z \). Other people messed up the very simple algebra. These people should review their algebra! BTW, this was problem 32 from section 12.5 that I did it in class (by accident, I picked a random problem and forgot that this was the problem that I picked for the “quiz”).