1. Find an equation for the tangent plane to the parametric surface

\[ x = v^2, \quad y = u + v, \quad z = u^2, \]

at the point \((1, 2, 1)\). Simplify as much as you can!

**Sol.** Here

\[ \mathbf{r}(t) = \langle v^2, u + v, u^2 \rangle \]

Taking derivatives with respect to \(u\) and \(v\), we get

\[ \mathbf{r}_u = \langle 0, 1, 2u \rangle, \quad \mathbf{r}_v = \langle 2v, 1, 0 \rangle. \]

Next, we have to find out what are \(u\) and \(v\) at the point \((1, 2, 1)\). We have to solve, for \(u, v\):

\[ 1 = v^2, \quad 2 = u + v, \quad 1 = u^2 \]

From the first equation \(v = -1\) or \(v = 1\), from the last, \(u = -1\) or \(u = 1\), but to satisfy the second equation, only \(u = 1\) and \(v = 1\) are OK. So we know that at the designated point, \(u = 1, v = 1\).

Plugging these above gives:

\[ \mathbf{r}_u = \langle 0, 1, 2 \rangle, \quad \mathbf{r}_v = \langle 2, 1, 0 \rangle. \]

To find the normal, we take the cross-product

\[ \mathbf{n} = \langle 0, 1, 2 \rangle \times \langle 2, 1, 0 \rangle = \langle -2, 4, -2 \rangle. \]

(you do it!).

The equation of the tangent plane is

\[ \langle x - x_0, y - y_0, z - z_0 \rangle \cdot \mathbf{n} = 0, \]

So, in this problem, it is

\[ \langle x - 1, y - 2, z - 1 \rangle \cdot \langle -2, 4, -2 \rangle = 0, \]

that spells out to:

\[ (-2)(x - 1) + 4(y - 2) + (-2)(z - 1) = 0. \]
Dividing both sides by 
−2 and simplifying, we get

\[ x - 2y + z = -2 \] .

**Ans.** \( x - 2y + z = -2 \) (type: Eq. of a plane).

**Comments:** About %40 got it perfectly, another %20 got it correctly, but didn’t completely simplify, another %20 did it the right way but messed up somewhere. Some people did a very bad mistake, by not plugging in \( u = 1, v = 1 \). You had to find what \( u \) and \( v \) are at the designated point, and then plug-them-in. If you are not sure how to find the \( u \) and \( v \) (like I did above), you should confess, and stop right there. Leaving \( u \) and \( v \) in the answer is **nonsense**!

2. Evaluate the surface integral

\[ \iint_S z \, dS \]

where \( S \) is the triangular region with vertices \((2, 0, 0), (0, 2, 0), (0, 0, 2)\).

**Sol.** We first find the equation of the plane passing through the three points. This turns out to be

\[ x + y + z = 2 \] .

(in this easy case you can do it by “inspection” (adding up the three coordinates always gives you 2, in general you would have to work hard, doing \( \mathbf{n} = \mathbf{AB} \times \mathbf{AC} \) etc.)

Expressing this plane in **explicit** form, we have

\[ z = 2 - x - y \] .

The relevant formula is:

\[ \iint_S f(x, y, z) \, dS = \iint_D f(x, y, g(x, y)) \sqrt{1 + g_x^2 + g_y^2} \, dx \, dy \] ,

where \( D \) is the projection of the region on the \( xy \)-plane.

Here \( g(x, y) = 2 - x - y \), so \( g_x = -1, g_y = -1 \), and \( \sqrt{1 + g_x^2 + g_y^2} = \sqrt{3} \). So

\[ \iint_S z \, dS = \iint_D (2 - x - y) \sqrt{3} \] .

It still remains to find out the region \( D \). The plane \( z = 2 - x - y \) meets the \( xy \) plane (alias \( z = 0 \)) at the line \( x + y = 2 \). Since \( x \geq 0, y \geq 2 \) the region \( D \) is

\[ D = \{(x, y)|x \geq 0, y \geq 0, x + y \leq 2 \} \] .

A type I description is

\[ D = \{(x, y)|0 \leq x \leq 2, 0 \leq y \leq 2 - x \} \] .
So we get 
\[ \int_0^2 \int_0^{2-x} \sqrt{3} \, dy \, dx . \]

The inner integral is
\[ \int_0^{2-x} \sqrt{3} \, dy = \sqrt{3}y \bigg|_0^{2-x} = \sqrt{3}(2-x) . \]

The outer integral is:
\[ \int_0^2 \sqrt{3}(2-x) \, dx = \sqrt{3}(2x - \frac{x^2}{2}) \bigg|_0^2 = 2\sqrt{3} . \]

**Ans.:** $2\sqrt{3}$ (type: number).

**Comments:** I really didn’t allow enough time, so no one got it completely. Quite a few courageous people almost got it, but only messed up in figuring out $D$, and took it as $[0, 2] \times [0, 2]$. 