NAME: **** Section: ****

MATH 251 (1-6,10-11), Dr. Z. , Third Practice Test for Exam #2

Do not write below this line (office use only)

1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)

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total:

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1
1. (10 points) By finding a function $f$ such that $\mathbf{F} = \nabla f$, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve $C$.

$$\mathbf{F}(x, y, z) = xe^{x^2+y^2+z^2} \mathbf{i} + ye^{x^2+y^2+z^2} \mathbf{j} + ze^{x^2+y^2+z^2} \mathbf{k},$$

$$C: x = 2t \quad , \quad y = t^2 \quad , \quad z = t^5 \quad , \quad 0 \leq t \leq 1.$$ 

Ans:
2. (10 points) (a) Find a function $f$ such that $\mathbf{F} = \nabla f$ and (b) use part (a) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve $C$.

$$\mathbf{F}(x, y, z) = (2xz + y^2) \mathbf{i} + 2xy \mathbf{j} + (x^2 + 3z^2) \mathbf{k}$$,

$$C: x = t^2, \quad y = t + 1, \quad z = 2t - 1, \quad 0 \leq t \leq 1$$.

Ans.: 3
3. (10 points) Evaluate
\[ \iiint_{E} (x^2 + y^2 + z^2)^9 \, dV \]
where \( E \) is bounded by the xy-plane and the hemispheres \( z = -\sqrt{1 - x^2 - y^2} \) and \( z = -\sqrt{9 - x^2 - y^2} \).

\[ \text{Ans.: } 4 \]
4. (10 points) Plot the vector field \( \mathbf{F} = \langle -x, -y \rangle \).

Ans.: 
5. (10 points) Find the Jacobian of the transformation from \((u, v, w)\)-space to \((x, y, z)\)-space.

\[
x = u^3 + v^2 + w, \quad y = u^2 + v, \quad z = u + v + w,
\]

at the point \((u, v, w) = (1, 2, 3)\).

Ans.: 6
6. (10 points) Evaluate the iterated integral by converting to polar coordinates.

\[
\int_{0}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} x^2 y \, dx \, dy
\]

Ans.:
7. (10 points) Calculate the iterated integral

\[ \int_0^1 \int_0^y (x^2y + xy^2) \, dx \, dy. \]

Ans.:
8. (10 points) Evaluate the triple integral

\[ \int \int \int_{E} x^2 y^2 \, dV , \]

where \( E \) is bounded by \( z = 1 - y^4 \) and the planes \( z = 0, x = 1 \) and \( x = -1 \).

\[ \text{Ans.} : \]

\[ \text{Ans.} : \]
9. (10 points) Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = (9 + xy)(x + y)$

local maximum value(s):

local minimum value(s):

saddle point(s):
10. (10 points) Evaluate the iterated integral

\[ \int_0^1 \int_x^{2x} \int_0^y 2xyz \, dz \, dy \, dx \ . \]

Ans.: 11