MATH 251 (1-6,10-11 ), Dr. Z. , Third Practice for Exam 1 (Version of Oct. 7, 2009, 4:28pm, thanks to Britanie Peabody)
(Previous version Oct. 6, 2009, 1:20pm, thanks to Jonathan Liu!)

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
Do not write below this line

1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)

Types: Number, Function of variable(s), 2D vector of numbers, 3D vector of numbers, 2D vector of functions, 3D vector of functions, equation of a plane, parametric equation of a line, equation of a line, equation of a surface, equation of a line, DNE (does not exist).
1. (10 points) Use the chain rule to find $f_u$ and $f_v$ if

$$ f(x, y) = x^3 + y^3 \quad , \quad x = e^{u+v} \quad , \quad y = 2u + 3v \quad . $$

Express your answer in terms of $u$ and $v$.

The **types** of the answer are:
2. (10 points) Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$

$$f(x, y) = x^3 + xyz + y^3 + z^3,$$

and the variables are related by $x = r^2 + 2s$ and $y = 3r + 2s^2$. You can leave your answer in terms of $x, y, z$ as well as $r, s$, and you do not need to simplify!

The type of the answers is:
3. (10 points) Find the maximal rate of change of $f(x, y, z) = x^2y^3z^4$ at (1, 1, 1), and the unit direction where it occurs.

The types of the answers are:
4. (10 points) Find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) if

\[
x^3 + y^3 + e^{x+y+z} = 5 \cos(x + y + z) + 1.
\]

The types of the answer are:
5. (10 points) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$z^3 + x^3 + y^3 + 3xyz = 6.$$ 

The types of the answer are:
6. (10 points) Find the limit if it exists, or show that the limit does not exist.

\[
\lim_{(x,y)\to(0,0)} \frac{5x + 2y}{2x + 3y + 1}.
\]

The type of the answer is:
7. (10 points) A certain particle has law of motion

\[ \mathbf{r}(t) = (\sin t, \cos 2t, e^t) \]

Find its velocity, acceletartin, and speed at \( t = \pi/6 \).

The **types** of the answers are:
8. (10 points) Find the limit if it exists, or show that the limit does not exist.

\[
\lim_{(x,y) \to (0,0)} \frac{x^{10}}{(x^2 + y^2)^2}.
\]

The type of the answer is:
9. (10 points, altogether) Do the following limits exist? If they do, find them. Explain!

The types of the answers are: and .

a. (5 points) If

\[
\lim_{(x,y,z) \to (1,2,3)} f(x,y,z) = 1 , \quad \lim_{(x,y,z) \to (1,2,3)} g(x,y,z) = 2
\]

compute

\[
\lim_{(x,y,z) \to (1,2,3)} (f(x,y,z) + g(x,y,z))^3 e^{g(x,y,z)}
\]

b. (5 points)

\[
\lim_{(x,y) \to (0,0)} \frac{x^4 + y^4}{2x^4 + y^4}
\]
10. (10 pts.) Compute the partial derivatives with respect to $x$ and $y$. 

$$z = xy \ln(x^3 + xy + y^3)$$

The types of the answers are:
Answers: 1. $f_u = 3e^{3(u+v)} + 6(2u + 3v)^2$, $f_v = 3e^{3(u+v)} + 9(2u + 3v)^2$. (Type: functions of $u,v$).

2. $f_r = 2(3x^2 + yz)r + 3(xz + 3y^2)$, $f_s = 2(3x^2 + yz) + 4s(xz + 3y^2)$. (Type: functions).

3. $\sqrt{29}, \left\langle \frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}} \right\rangle$ (types: number, vector of numbers).

4. $\frac{\partial z}{\partial x} = -\frac{3x^2 + e^{x+y} + z + 5\sin(x+y+z)}{e^{x+y} + 5\sin(x+y+z)}, \frac{\partial z}{\partial y} = -\frac{3y^2 + e^{x+y} + z + 5\sin(x+y+z)}{e^{x+y} + 5\sin(x+y+z)}$. (Type: functions of $x,y$ (but also using $z$, that is an implicitly defined function of $(x,y)$)).

5. $\frac{\partial z}{\partial x} = -\frac{3x^2 + 3yz}{3z^2 + 3xy}, \frac{\partial z}{\partial y} = -\frac{3y^2 + 3xz}{3z^2 + 3xy}$. (Type: functions of $x,y$ (but also using $z$, that is an implicitly defined function of $(x,y)$)).

6. 0 (type: Number).

7. $v = \left\langle \frac{\sqrt{3}}{2}, -\sqrt{3}, e^{\pi/6} \right\rangle$, $a = \left\langle -\frac{1}{2}, -2, e^{\pi/6} \right\rangle$, speed = $\sqrt{\frac{15}{4} + e^{\pi/3}}$ (types: vector of numbers, vector of numbers, positive number).

8. 0 (type: Number). [Use Polar coordinates].

9. (a) $27e^2$ (type: Number). (b): DNE (type: DNE).

10. 

$$\frac{\partial z}{\partial x} = y \ln(x^3 + xy + y^3) + \frac{xy(3x^2 + y)}{x^3 + xy + y^3}$$

$$\frac{\partial z}{\partial y} = x \ln(x^3 + xy + y^3) + \frac{xy(x + 3y^2)}{x^3 + xy + y^3}$$