NAME: (print!) _____

Section: ____ E-Mail address: _____

MATH 251 (1-6,10-11), Dr. Z. , Second Practice for Exam 1 (Version of 6:36pm, Oct. 6, 2009, thanks to Marty L.)

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM Do not write below this line

1. (out of 10)

- $2. \qquad (\text{out of } 10)$
- $3. \qquad (\text{out of } 10)$
- 4. (out of 10)
- 5. (out of 10)
- 6. (out of 10)
- 7. (out of 10)
- $8. \qquad (out of 10)$
- 9. (out of 10)
- 10. (out of 10)

Types: Number, Function of *variable*(s), 2D vector of numbers, 3D vector of numbers, 2D vector of functions, 3D vector of functions, equation of a plane, parametric equation of a line, equation of a surface, equation of a line, DNE (does not exist).

1. (10 pts.) Find, as a function of t, the curvature of the curve

$$\mathbf{r}(t) = \langle e^t, t, e^{2t} \rangle$$

2. (10 points) Find $\frac{\partial h}{\partial r}$ at (q,r) = (3,2) where $h(u,v) = ve^u$, $u = q^2$, $v = q^2r$

The \mathbf{type} of the answer is:

3. (10 points) Find $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ as a function of t if $\mathbf{r}(t) = \langle t, \cos t, \sin t \rangle$.

The \mathbf{type} of the answer is:

4. (10 points) Find an equation of the tangent plane at the given point

$$z = xy\ln(x+y) \quad , (1,1) \quad .$$

5. (10 points) Find $\frac{\partial f}{\partial s}$ at (r,s) = (1,0), where $f(x,y) = \ln(xy)$, x = 3r + 2s, and y = 5r + 3s.

6. (10 points) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$x^3 + y^3 + z^3 = 2xyz + 1 \quad .$$

7. (10 points) Describe the vertical and horizontal traces of $f(x, y) = 9 - x^2 - y^2$. Sketch a few of them.

type of answers: Families of curves in xy-plane, xz-plane, and yz-plane. Diagrams.

8. (10 points) Find the length of the curve

$$\mathbf{r}(t) = \langle 1, 3e^t, 4e^t \rangle \quad 0 \le t \le \ln 3 \quad .$$

9. (10 points) Which of the following is the arc length parametrization of a circle of radius 10 centered at (1, 2). Explain!

- (a) $\mathbf{r_1}(t) = \langle 1 + 10 \cos t, 2 + 10 \sin t \rangle$
- (b) $\mathbf{r_2}(t) = \langle 1 + 10 \cos 10t, 2 + 10 \sin 10t \rangle$
- (c) $\mathbf{r_3}(t) = \langle 1 + 10 \cos \frac{t}{10}, 2 + 10 \sin \frac{t}{10} \rangle$
- (d) $\mathbf{r_4}(t) = \langle 1 + 10 \cos 5t, 2 + 10 \sin 5t \rangle$
- (e) $\mathbf{r_5}(t) = \langle 10 \cos t, 10 \sin t \rangle$

10. (10 points) Find an equation to the plane that passes through the points (1, 2, 3), (2, 1, 3), (3, 2, 1).

The \mathbf{type} of the answer is:

Answers

- 1. $\frac{e^t \sqrt{1+16e^{2t}+4e^{4t}}}{(\sqrt{1+e^{2t}+4e^{4t}})^3}$ (type: Function of t).
- 2. $9e^9$ (type: Number). [Thanks to Sammy G. for the correction!]
- 3. 0 and 1 (types: functions of t that happen to be constant functions (by accident)).
- 4. $z = (\ln 2 + \frac{1}{2})x + (\ln 2 + \frac{1}{2})y (\ln 2 + 1)$ (type: Equation of Plane).
- 5. $\frac{19}{15}$ (type: Number).
- 6. $-\frac{3x^2-2yz}{3z^2-2xy}$, $-\frac{3y^2-2xz}{3z^2-2xy}$ (type: functions).

7. $x^2 + y^2 = 9 - c$ (type: family of circles in *xy*-plane); $z = 9 - c^2 - y^2$ (type: family of parabolas in *yz*-plane); $z = 9 - c^2 - x^2$ (type: family of parabolas in *xz*-plane);

- 8. 10 (type: Number).
- 9. (c) (because the magnitute of $\mathbf{r_3}'(t)$ is always 1.
- 10. x + y + z = 6 (type: Equation of plane).