Answers to Spring 2012 Math 152 Final

Disclaimer: Not responsible for any errors. One dollar prize for the **first** discoverer of any error.

Version of Dec. 15, 2012, 10:35am: thanks to Eunhee Kim, correcting 9(c).

Version of Dec. 15, 2012, 10:27am: thanks to Eunhee Kim, correcting 10(a) (the r^2 was before r).

Version of Dec. 15, 2012, 9:55am: thanks to Taylor Picillo, correcting the limits of integration in 7(b) (abd hence the value of the integral).

Version of Dec. 11, 2012, 7:40am, thanks to James Chico, correcting a typo in 6(b)

1. (a) $\frac{1}{2}x\sqrt{x^2+1} - \frac{1}{2}\ln(x+\sqrt{1+x^2}) + C$

[Note added Dec. 10, 2012: the previous answer was correct, but unsimplified, it was the solution that Maple gave. So humans are still (sometimes!) superior. I thank Diego Vega for pointing this out]

(b) $x^8(\frac{1}{2}\ln x - \frac{1}{16}) + C$ (Hint: First use **algebra** to write $\ln x^4$ as $4\ln x$, then integrate by parts).

2. (a) cond. conv. (b) abs. conv. (by straight comparison test: $2^n + \sqrt{n} > 2^n$ entails $\frac{1}{2^n+\sqrt{n}} < \frac{1}{2^n}$, and then the geometric series test (or ratio test, or root test). (c) div. (by the divergence test, the limit of the **sequence** is $\frac{1}{2}$, and is not 0). (d) div. (by ratio test, or root test, $\rho = \frac{3}{2} > 1$).

3. By integration by parts (twice!, also for $\int(\cos \alpha x)\, e^{-sx}\, dx$, and then using algebra)

$$\int (\sin \alpha x)\, e^{-sx}\, dx = -\frac{\alpha}{s^2+\alpha^2} e^{-sx} \cos \alpha x - \frac{s}{s^2+\alpha^2} e^{-sx} \sin \alpha x$$

Now find $\int_0^R \sin \alpha x e^{-sx}\, dx$ and take the limit as $R \to \infty$, using the "sandwich theorem" from calc1, (sine and cosine are always between -1 and 1 and $\lim_{R\to\infty} e^{-R} = 0$, and of course, since $s > 0$, $\lim_{R\to\infty} e^{-sR} = 0$).

Comment: This is a **very hard problem** and this type is unlikely to show up in this semester's final, but one never knows.

4. (a) $\frac{1}{2}$ (b) conv. (by root test, $\rho = \frac{1}{2} < 1$) ; div. (by root test, $\rho = \frac{3}{2} > 1$) ; conv. (by root test, $\rho = \frac{\sqrt{2}}{2} < 1$) ;

5. (a) $y = e^{1-e^{-t}}$

(b) $r \leq 3$, $\rho \sin\theta \leq 3$ [Comment, this topic will not be covered this semester, due to Sandy]

6. (a) $1 - x^2 + x^4/2! + \ldots + (-1)^n \frac{x^{2n}}{n!} + \ldots$ (or $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$)

(b) $\sum_{n=1}^{\infty}(-1)^n \frac{(0.4)^{2n}}{n!(2n)}$

(c) five terms (but very hard without a calculator)

7. (a) $\frac{4}{3}$ (b) $\pi \int_{-\sqrt{3}}^{\sqrt{3}} (2-\sqrt{1+x^2})^2\,dx = \pi\left(4\sqrt{3}-4\ln(2-\sqrt{3})\right)$

[Comment, the integral is rather tedious, first algebra, then trig. substitution to do $\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{1+x^2}\,dx$, but this may have been in the formula sheet last semester]

8. (a) $y = 1$ (b) $2\pi \int_0^5 (2t)\sqrt{5}\,dt$ $(= 50\sqrt{5}\pi)$.

9. (b) $x = e^{-2\theta}\cos\theta$, $y = e^{-2\theta}\sin\theta$; (c) $\frac{\sqrt{5}}{2}$

10. (a) $r^2 = \sec 2\theta$

(b) $\frac{1}{2}\int_{-\cos^{-1}\frac{1}{4}}^{\cos^{-1}\frac{1}{4}} (16\cos^2\theta - 1)\,d\theta$