

NAME: (print!) _____

Section: _____ E-Mail address: _____

MATH 152 (01-03, 07-09), Fall 2012, Dr. Z.'s , Second Practice Final

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question) (when applicable)

Explain your work! Do not write below this line

1. (out of 25)
2. (out of 10)
3. (out of 15)
4. (out of 10)
5. (out of 15)
6. (out of 10)
7. (out of 15)
8. (out of 10)
9. (out of 15)
10. (out of 10)
11. (out of 15)
12. (out of 10)
13. (out of 15)
14. (out of 10)
15. (out of 15) _____
- tot. (out of 200)

Warning: The Final Exam is composed by Prof. Scheffer, and may look different than this one. For example, in the real final exam, some problems are worth more than others, and some have more parts.

1. (25 pts) (a) Consider the region in the xy -plane bounded by $y = x^2(1 + x^4)^{-1}$, the x -axis, and $x = 1$. Find the volume of the solid obtained by rotating the region about the y -axis.
- (b) Consider the region in the xy -plane bounded by $y = \sqrt{\sin^{-1} x}$, the x -axis, and $x = 1/2$. Find the volume of the solid obtained by rotating the region about the x -axis. Recall that $\sin^{-1} x$ is $\arcsin x$.

2. (10 points)

Decide whether the infinite series

$$\sum_{n=3}^{\infty} \sqrt{\frac{n^2 - 3}{n^4 + n^3 + 4}}$$

is absolutely convergent, conditionally convergent, or divergent. Explain!

3. (15 points)

Decide whether the infinite series

$$\sum_{n=3}^{\infty} \frac{(2n)!}{n!^2 5^n}$$

is absolutely convergent, conditionally convergent, or divergent. Explain!

4. (10 points) An infinite sequence is defined by $a_1 = 1.4$, and $a_{n+1} = \sqrt{12 + a_n}$, for $n = 1, 2, 3, \dots$. Assume that the sequence converges. Find $\lim_{n \rightarrow \infty} a_n$.

5. (15 points) Find the first four nonzero terms of the Maclaurin series for $(1 + x^2) \cos x$.
Do not differentiate $(1 + x^2) \cos x$.

6. (10 points) Evaluate the improper integral $\int_0^\infty \frac{dx}{9+x^2}$.

7. (15 points) Find the first four nonzero terms of the Maclaurin series for $(9 + x^2)^{-1}$. Do not differentiate $(9 + x^2)^{-1}$.

8. (10 points) Find all solutions of the differential equation $\frac{dy}{dx} = x^4(1 - y^2)^{1/2}$.

9. (15 points) Find the interval of convergence of the power series

$$\sum_1^{\infty} \frac{2^n (x-2)^n}{\sqrt{n+1}}$$

10. (10 points) Find the **sum** of the infinite series $\sum_{n=2}^{\infty} \frac{2^n + 4^n}{5^n}$.

11. (15 points) Find the area of the region in the xy -plane which is bounded by $r = (\cos \theta)(\sin \theta)^{3/2}$, $\theta = \pi/6$, $\theta = \pi/3$ in polar coordinates.

12. (10 points) Consider the function $f(x)$ with the property

$$f^{(3)}(x) = \frac{1}{x^2 + 5} \quad \text{for all real numbers } x.$$

Let $T_2(x)$ be the second Taylor polynomial of $f(x)$ centered at the point $a = 3$. Use the Error Bound to find a bound for $|f(5) - T_2(5)|$.

13. (15 points) Find the length of the curve $y = \ln(\sin x)$, $\pi/4 \leq x \leq \pi/3$.

14. (10 points) Evaluate $\int \tan^4 x \, dx$.

15. (15 points) Evaluate $\int \frac{2x^2-x-3}{(x^2+1)(x-1)} dx$.