NAME: (print!) ____

Section: ________ **E-Mail address:** ________________________

MATH 152 (01-03, 07-09), Fall 2012, Dr. Z.'s , Practice Final

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question) (when applicable)

Version of Dec. 10, 2012, 11:00am, correcting 2(a), thanks to Eunhee Kim, Tiffany Rodriguez, and Soojung Park (who share one dollar), the previous version was too complicated, due to a typo.

Explain your work! Do not write below this line

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1. (out of 14)

2. (out of 14)

3. (out of 14)

4. (out of 14)

5. (out of 14)

6. (out of 14)

7. (out of 14)

8. (out of 14)

9. (out of 14)

10. (out of 14)

11. (out of 14)

12. (out of 14)

13. (out of 14)

14. (out of 14)

15. (out of 16)

16. (out of 16)

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tot. (out of 200)

Warning: The Final Exam is composed by Prof. Scheffer, and may look different than this one. For example, in the real final exam, some problems are worth more than others, and some have more parts.

1. (14 pts) (a) Sketch (roughly) the region R bounded by the curves $y = x^2 + 1$ and $y = 19 - x^2$

(b) Find the area of R (Give exact answer, show work)

(c) Set-up but do not evaluate an integral for the volume which results when the region R is revolved about the x-axis.

(d) Set-up but do not evaluate an integral for the volume which results when the region R is revolved around the line $x = 4$.

Answers: (b)

(c)

(d)

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2. (14 pts, 7 each)

(a)
[version of Dec. 10, 2012, correcting a typo]
Evaluate the integral

$$\int \frac{2x^2 - 2x + 9}{(x-2)\,(x^2+9)}\,dx$$

(b) Evaluate the integral

$$\int_0^4 \sqrt{16 - x^2}\,dx \quad .$$

Ans. (a) **(b)**

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3. (14 pts, 7 each)

(a) Evaluate

$$\int_0^{\pi/2} \sin^2 x \cos^7 x \, dx \quad .$$

(b) Evaluate

$$\int x^2 e^{-9x} \, dx \quad .$$

Ans. (a) (b)

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4. (14 pts) Solve the initial value problem

$$\frac{dy}{dx} = \frac{-3\sin 3x}{y} \quad , y(0) = 1 \quad .$$

Express your answer explicitly, not implicitly, as a function of x.

Ans. $y =$

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5. (14 pts) Find the length of the curve

$$x = e^{4t} - 4t \quad , \quad y = 4e^{2t} \quad , 0 \leq t \leq 2 \quad .$$

Give an exact answer in terms of well-known constants like e and π, not a decimal approximation.

Ans.

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6. (14 pts, [(a) 6 and (b)8]) The curve

$$x = 1 - t^2 \quad , \quad y = \sin \pi t \quad , \quad -2 \leq t \leq 2$$

crosses itself at the origin $(0, 0)$.

(a) What are the t-values at which it crosses the origin

(b) Find the equations of both tangent lines at the origin.

Ans. (a) $t =$ **and** $t =$

(b) **and**

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7. (14 pts, 7 each) For each of the following series, decide whether the series converges absolutely, converges conditionally, or diverges. In each case state the convergence test used and show the method used.

(a)

$$\sum_{n=1}^{\infty} \frac{4\sqrt{n}}{n^3+9}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3}$$

Ans. (a) (b)

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8. (14 pts, 7 each) For each of the following series, decide whether the series converges absolutely, converges conditionally, or diverges. In each case state the convergence test used and show the method used.

(a)

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2}{n^{1/n}}$$

(b)

$$\sum_{n=1}^{\infty} \frac{1+3^n}{5^n}$$

Ans. (a)

(b)

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9. (14 pts) How many terms of the series

$$\sum_{1}^{\infty} \frac{(-1)^n}{n^3}$$

are required to approximate the sum of the series to within 0.001? Use the alternating-series error estimate, and show your reasoning.

Ans.

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10. (14 pts, [5,5,4]) Suppose that $\{a_n\}_{n=1}^{\infty}$ **is a convergent sequence with** $\lim_{n\to\infty} a_n = 4$ **and** $\{b_n\}_{n=1}^{\infty}$ **is a convergent sequence with** $\lim_{n\to\infty} b_n = 5$**.**
Which of the following sequences are convergent, and for those that are, find the limits.

$$(a)\{2a_n + 3b_n\}_{n=1}^{\infty}$$

$$(b)\{\frac{a_n}{b_n}\}_{n=1}^{\infty}$$

$$(c)\{\frac{a_n}{b_n - 5}\}_{n=1}^{\infty}$$

Ans. (a) **(b)** **(c)**

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11. (14 pts, 7 each) What is the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x)^n}{n+8}$? For which values of x does the series converge?

Ans. (a) radius of convergence=

(b)

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12. (14 pts) Without attempting to compute it, decide whether the improper integral

$$\int_1^{\infty} e^{-x^4}\, dx$$

converges or diverges. Show your reasoning.

Ans.

13. (14 pts, [5,5,4])

(a) Use the Maclaurin series for e^x to get an infinite series for the integral

$$\int_0^1 e^{-\frac{1}{3}x^3}\,dx \quad .$$

Your answer should include an expression for the n-th term of the series.

(b) Use the first two terms of your series to get an approximate value for I

(c) Estimate the error in your approximation, showing your reasoning.

Ans. (a)

(b)

(c)

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14. (14 pts [9 for (a) and 5 for (b)]) Let

$$I = \int_1^5 \frac{1}{x} \quad .$$

Reminders:

$$S_N = \frac{1}{3}\Delta x \left[y_0 + 4y_1 + 2y_2 + \ldots + 4y_{N-3} + 2y_{N-1} + 4y_{N-1} + y_N \right] \quad ,$$

where $\Delta x = \frac{b-a}{N}$**, and** $y_j = f(a + j\Delta x)$**. Also recall**

$$Error(S_N) \leq \frac{K_4(b-a)^5}{180N^4} \quad ,$$

where K_4 **is a number that that** $|f^{(4)}(x)| \leq K_4$ **for all** $x \in [a, b]$**.**

(a) Use Simpson's rule with $N = 4$ **subdivisions to find an approximation, call it** J**.**

Ans to (a)

(b) Use the error estimate to find an upper bound for the error $|I - J|$**.**

Ans to (b)

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15. (16 pts, [8,4,4])

(a) Find the second degree Taylor polynomial $T_2(x)$ for $f(x) = x^{10}$ about $a = 1$.

(b) Use this polynomial to approximate $(1.01)^{10}$

(c) Using the error estimate

$$|T_n(x) - f(x)| \leq K \frac{|x-a|^{n+1}}{(n+1)!} \quad ,$$

where K is a number such that $|f^{(n+1)}(x)| \leq K$ for all u between a and x, estimate the error.

Ans. (a)

(b)

(c)

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16. (16 pts) Use known Maclaurin series to find the Maclaurin polynomial of degree 3 of $f(x) = e^x \cos(\sin x)$.

Ans.
