Solutions to Attendance Quiz #4 for Dr. Z.'s Calc2 for Sept. 24, 2012

1. Evaluate the integral

$$\int x \sin x \; dx$$

Sol. of 1: Start with a blank table

$$u = \qquad\qquad u' =$$

$$v = \qquad\qquad v' =$$

It is reasonable to take u to be x, and hence v' to be $\sin x$. Now our table looks like:

$$u = x \qquad\qquad u' = 1$$

$$v = -\cos x \qquad\qquad v' = \sin x$$

Putting the above data into the **Integration-by-parts** formula

$$\int uv' \, dx \; = \; uv - \int u'v \, dx \quad ,$$

we get

$$\int x \sin x \; dx \; = \; x(-\cos x) - \int 1 \cdot (-\cos x) \; dx =$$

$$-x\cos x \; + \int \cos x \; dx = -x\cos x + \sin x + C$$

Ans to 1: $-x\cos x + \sin x + C$.

Comment: About %65 of the students got the right answer. Most other students did it essentially the right way, but messed up the set-up, or the algebra (sign mistakes) or the fact that the anti-derivative of $\sin x$ is $-\cos x$ (some people forgot the minus sign).

2. Evaluate the integral

$$\int x^2 e^x \; dx$$

Sol. of 2: Start with a blank table

$$u = \qquad\qquad u' =$$

$$v = \qquad\qquad v' =$$

It is reasonable to take u to be x^2, and hence v' to be e^x. Now our table looks like:

$$u = x^2 \qquad\qquad u' = 2x$$

$$v = e^x \qquad\qquad v' = e^x$$

Putting the above data into the **Integration-by-parts** formula

$$\int uv'\,dx \;=\; uv - \int u'v\,dx \quad ,$$

we get

$$\int x^2e^x \;dx \;=\; x^2e^x - \int (2x)\cdot e^x \;dx \quad .= x^2e^x - 2\int xe^x \;dx \quad .$$

We are now faced with a **subproblem**, to find $\int xe^x$. We also use *Integration by Parts* with $u = x, v' = e^x$, giving $u' = 1, v = e^x$, and

$$\int xe^x\,dx = xe^x - \int (1)\cdot(e^x)\,dx = xe^x - \int e^x\,dx = xe^x - e^x = e^x(x-1) \quad .$$

Going back to the **main** problem, we get

$$\int x^2e^x \;dx \;= x^2e^x - 2\int xe^x \;dx = x^2e^x - 2e^x(x-1) = e^x(x^2-2x+2) + C \quad .$$

Ans to 2: $e^x(x^2 - 2x + 2) + C$.

Comment: About %50 of the students got the right answer. Most other students did it essentially the right way, but messed up the algebra. Many people got as far as $x^2e^x - 2e^x(x-1)$ but then messed-up the algebra. **PLEASE** review your basic algebra skills, how to open-up parantheses correctly and to factor-out correctly!

3. Evaluate the integral

$$\int x \ln x \;dx$$

Sol. of 3: Start with a blank table

$$u = \qquad\qquad u' =$$

$$v = \qquad\qquad v' =$$

It is reasonable to take u to be $\ln x$, and hence v' to be x. Now our table looks like:

$$u = \ln x \qquad\qquad u' =$$

$$v = \qquad\qquad v' = x$$

Completing the table, we get:

$$u = \ln x \qquad\qquad u' = \frac{1}{x}$$

$$v = \frac{x^2}{2} \qquad\qquad v' = x$$

Putting the above data into the **Integration-by-parts** formula

$$\int uv'\,dx \;=\; uv - \int u'v\,dx \quad ,$$

we get

$$\int x \sin x \;dx \;=\; (\ln x)(\frac{x^2}{2}) - \int \frac{1}{x} \cdot \frac{x^2}{2}\,dx$$

$$= (\ln x)(\frac{x^2}{2}) - \int \frac{x}{2}\,dx = (\ln x)(\frac{x^2}{2}) - \frac{x^2}{4} + C$$

Ans to 3: $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

Comment: About %75 of the students got the right answer. Another %15 did it the right way, but messed up the set-up, and about %10 were clueless.