Solutions to Attendance Quiz #4 for Dr. Z.’s Calc2 for Sept. 24, 2012

1. Evaluate the integral

\[ \int x \sin x \, dx \]

**Sol. of 1:** Start with a blank table

\[
\begin{array}{c|c}
   u       & u' = \\
   v       & v' = \\
\end{array}
\]

It is reasonable to take \( u = x \), and hence \( v' = \sin x \). Now our table looks like:

\[
\begin{array}{c|c}
   u = x       & u' = 1 \\
   v = -\cos x & v' = \sin x \\
\end{array}
\]

Putting the above data into the **Integration-by-parts** formula

\[
\int uv' \, dx = uv - \int u'v \, dx ,
\]

we get

\[
\int x \sin x \, dx = x(-\cos x) - \int 1 \cdot (-\cos x) \, dx =
\]

\[-x \cos x + \int \cos x \, dx = -x \cos x + \sin x + C\]

**Ans to 1:** \(-x \cos x + \sin x + C\).

**Comment:** About 65% of the students got the right answer. Most other students did it essentially the right way, but messed up the set-up, or the algebra (sign mistakes) or the fact that the anti-derivative of \( \sin x \) is \(-\cos x\) (some people forgot the minus sign).

2. Evaluate the integral

\[ \int x^2 e^x \, dx \]

**Sol. of 2:** Start with a blank table

\[
\begin{array}{c|c}
   u =       & u' = \\
   v =       & v' = \\
\end{array}
\]
It is reasonable to take $u$ to be $x^2$, and hence $v'$ to be $e^x$. Now our table looks like:

\[
\begin{align*}
u &= x^2 & u' &= 2x \\
v &= e^x & v' &= e^x
\end{align*}
\]

Putting the above data into the **Integration-by-parts** formula

\[
\int uv' \, dx = uv - \int u'v \, dx ,
\]

we get

\[
\int x^2 e^x \, dx = x^2 e^x - \int (2x) \cdot e^x \, dx = x^2 e^x - 2 \int x e^x \, dx .
\]

We are now faced with a **subproblem**, to find $\int xe^x$. We also use **Integration by Parts** with $u = x, v' = e^x$, giving $u' = 1, v = e^x$, and

\[
\int xe^x \, dx = xe^x - \int (1) \cdot (e^x) \, dx = xe^x - \int e^x \, dx = xe^x - e^x = e^x(x - 1) .
\]

Going back to the **main** problem, we get

\[
\int x^2 e^x \, dx = x^2 e^x - 2 \int xe^x \, dx = x^2 e^x - 2e^x(x - 1) = e^x(x^2 - 2x + 2) + C .
\]

**Ans to 2:** $e^x(x^2 - 2x + 2) + C$.

**Comment:** About $50\%$ of the students got the right answer. Most other students did it essentially the right way, but messed up the algebra. Many people got as far as $x^2 e^x - 2e^x(x - 1)$ but then messed-up the algebra. **PLEASE** review your basic algebra skills, how to open-up parantheses correctly and to factor-out correctly!

3. Evaluate the integral

\[
\int x \ln x \, dx
\]

**Sol. of 3:** Start with a blank table

\[
\begin{align*}
u &= & u' &= \\
v &= & v' =
\end{align*}
\]

It is reasonable to take $u$ to be $\ln x$, and hence $v'$ to be $x$. Now our table looks like:

\[
\begin{align*}
u &= \ln x & u' &= \\
v &= & v' = x
\end{align*}
\]

Completing the table, we get:
\[ u = \ln x \quad u' = \frac{1}{x} \]
\[ v = \frac{x^2}{2} \quad v' = x \]

Putting the above data into the \textbf{Integration-by-parts} formula

\[ \int uv' \, dx = uv - \int u'v \, dx \]

we get

\[ \int x \sin x \, dx = (\ln x)(\frac{x^2}{2}) - \int \frac{1}{x} \, \frac{x^2}{2} \, dx \]

\[ = (\ln x)(\frac{x^2}{2}) - \int \frac{x}{2} \, dx = (\ln x)(\frac{x^2}{2}) - \frac{x^2}{4} + C \]

\textbf{Ans to 3:} \( \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C \)

\textbf{Comment:} About \%75 of the students got the right answer. Another \%15 did it the right way, but messed up the set-up, and about \%10 were clueless.