Solutions to Attendance Quiz #3 for Dr. Z.'s Calc2 for Sept. 20, 2012

1. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

$$y = 1/x^3 \quad , \quad y = 0 \quad , \quad x = 1 \quad , \quad x = 3$$

Sol. of 1: We use the formula

$$Volume \, = \, 2\pi \int_a^b x f(x) \, dx \quad .$$

with $f(x) = \frac{1}{x^3}$, $a = 1, b = 3$. So the volume is

$$Volume \, = \, 2\pi \int_1^3 x \frac{1}{x^3} \, dx \quad .$$

In order to compute the definite integral we **first** *simplify the integrand*, getting

$$Volume \, = \, 2\pi \int_1^3 \frac{1}{x^2} \, dx \quad .$$

Now

$$Volume \, = \, 2\pi \int_1^3 \frac{1}{x^2} = 2\pi \int_1^3 x^{-2} \, = \, 2\pi \frac{x^{-1}}{-1} \Big|_1^3 = -2\pi \frac{1}{x} \Big|_1^3$$

$$-2\pi \left(\frac{1}{3} - \frac{1}{1} \right) = -2\pi(-\frac{2}{3}) = \frac{4\pi}{3} \quad .$$

Ans. to 1: The volume is $\frac{4\pi}{3}$ cubic units.

Comment: %95 of the students set-it up correctly, and %80 got the final right answer.

2. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

$$y = 2 - x^2 \quad , \quad 4x + y = 5 \quad .$$

Sol. of 2: We first convert $4x + y = 5$ from *implicit* to *explicit* form, getting $y = 5 - 4x$.

We next have to find the **points of intersection** of $y = 2 - x^2$ and $y = 5 - 4x$.

Setting them equal gives

$$2 - x^2 = 5 - 4x \quad .$$

Moving everything to the left

$$2 - x^2 - 5 + 4x = 0 \quad .$$

Simplifying

$$-x^2 + 4x - 3 = 0 \quad .$$

Multiplying by -1:

$$x^2 - 4x + 3 = 0 \quad .$$

We have to solve this equation. Factorizing:

$$(x-1)(x-3) = 0 \quad ,$$

giving **two** roots, $a = 1$ and $b = 3$. We have to use the formula

$$Volume = 2\pi \int_1^3 x(TOP(x) - BOT(x))\, dx$$

Plugging-in $x = 2$ into $y = 2 - x^2$ gives -2 and into $y = 5 - 4x$ gives -3, so $y = 5 - 4x$ is the BOT, and $y = 2 - x^2$ is the TOP. So the integral is

$$Volume = 2\pi \int_1^3 x((2 - x^2) - (5 - 4x))\, dx = 2\pi \int_1^3 x(-3 + 4x - x^2)\, dx$$

$$= 2\pi \int_1^3 (-3x + 4x^2 - x^3)\, dx = 2\pi(-3x^2/2 + 4x^3/3 - x^4/4)\Big|_1^3$$

$$= 2\pi[(-3(3^2 - 1^2)/2 + 4(3^3 - 1^3)/3 - (3^4 - 1^4)/4) = 2\pi[-3(8)/2 + 4(26)/3 - (80)/4]$$

$$= 2\pi[-12 + 104/3 - 20] = 2\pi[104/3 - 32] = 2\pi[(104 - 96)/3] = 2\pi(8/3) = \frac{16\pi}{3} \quad .$$

Ans. to 2: The volume is $\frac{16\pi}{3}$ cubic units.

Comment: About %80 of the students set-it up correctly, but some people did not finish it, and other finished it and didn't get the right answer due to computational errors. About %40 of the students got the final correct answer.