Solutions to Attendance Quiz #1 for Dr. Z.'s Calc2 for Sept. 10 , 2012

1. Find the area of the region enclosed between $y = x^3 + 1$ and $y = 3x^2 - 2x + 1$

Solution of 1:

First Step: Find the points where the two graphs meet by setting them equal to each other, and solve for x.

We have

$$x^3 + 1 = 3x^2 - 2x + 1 \quad .$$

Moving all the stuff on the right to the left, we get

$$x^3 + 1 - (3x^2 - 2x + 1) = 0$$

Do the algebra (open parentheses)

$$x^3 + 1 - 3x^2 + 2x - 1 = 0$$

Simplify:

$$x^3 - 3x^2 + 2x = 0 \quad .$$

Factor x out

$$x(x^2 - 3x + 2) = 0 \quad .$$

Factor more

$$x(x-1)(x-2) = 0 \quad .$$

The three roots are $x = 0$, $x = 1$, $x = 2$.

Comment: About %90 of the people got this step correctly. Another %5 followed the method correctly, but messed up the algebra or factorization, and got other roots, that of course messed up everything else.

Step 2. We have two intervals of integration to worry about.

- (i) From $x = 0$ to $x = 1$
- (ii) from $x = 1$ to $x = 2$.

For each of these we must decide who is on **top** and who is on **bottom**.

For (i), pick $x = \frac{1}{2}$ and get that if you plug-it in into $y = x^3 + 1$ you get $\frac{9}{8}$ and if you plug-it-in $y = 3x^2 - 2x + 1$ you get $\frac{3}{4}$. Since $\frac{9}{8}$ is larger than $\frac{3}{4}$ we have

$$TOP = x^3 + 1 \quad , \quad BOT = 3x^2 - 2x + 1 \quad .$$

For (ii), pick $x = \frac{3}{2}$ and get that if you plug-it in into $y = x^3 + 1$ you get $\frac{35}{8}$ and if you plug-it-in $y = 3x^2 - 2x + 1$ you get $\frac{19}{4} = \frac{38}{4}$. Since $\frac{38}{4}$ is larger than $\frac{35}{8}$ we have

$$TOP = 3x^2 - 2x + 1 \quad , \quad BOT = x^3 + 1 \quad .$$

So the *set-up* is

$$\int_0^1 [(x^3 + 1) - (3x^2 - 2x + 1)]\, dx + \int_1^2 [(3x^2 - 2x + 1) - (x^3 + 1)]\, dx$$

Comment: About %80 of the people got the set-up correctly.

Step 3: Do the integration! In real life, you would use Maple or another software to get it right away. For example, in Maple you should type

```
int ((x**3+1)-(3x**2-2*x+1), x=0..1)+ int ((3x**2-2*x+1)-(x**3+1), x=1..2) ;
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and get the answer $\frac{1}{2}$ in a nano-second. But poor us, we have to do it by hand.

Let's do each integral separately.

First integral:

$$\int_0^1 [(x^3 + 1) - (3x^2 - 2x + 1)]\, dx \quad .$$

First, PLEASE simplify the integrand as much as possible. It is not a mistake to integrate right away, but takes longer and is more error prone. So the first step is to do:

$$\int_0^1 [(x^3 + 1) - (3x^2 - 2x + 1)]\, dx = \int_0^1 [x^3 - 3x^2 + 2x]\, dx \quad .$$

Now you integrate it piece-by-piece:

$$\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2}\Big|_0^1 = \frac{1^4}{4} - 3\frac{1^3}{3} + 2\frac{1^2}{2} - 0 = \frac{1}{4} - 1 + 1 = \frac{1}{4} \quad .$$

Second integral:

$$\int_1^2 [(3x^2 - 2x + 1) - (x^3 + 1)]\, dx \quad .$$

Again we simplify the integrand as much as possible.

$$\int_1^2 [(3x^2 - 2x + 1) - (x^3 + 1)]\, dx = \int_1^2 [-x^3 + 3x^2 - 2x]\, dx \quad .$$

Now you integrate it piece-by-piece:

$$-\frac{x^4}{4} + 3\frac{x^3}{3} - 2\frac{x^2}{2}\Big|_1^2 = \left[-\frac{2^4}{4} + 3\frac{2^3}{3} - 2\frac{2^2}{2}\right] - \left[-\frac{1^4}{4} + 3\frac{1^3}{3} - 2\frac{1^2}{2}\right]$$

$$= \left[-\frac{16}{4} + 3\frac{8}{3} - 2\frac{4}{2}\right] - \left[-\frac{1}{4} + 3\frac{1}{3} - 2\frac{1}{2}\right] = [-4 + 8 - 4] - \left[-\frac{1}{4} + 1 - 1\right] = 0 - (-\frac{1}{4}) = \frac{1}{4} \quad .$$

Adding up the two definite integrals, we get $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

Final Ans.: $\frac{1}{2}$.

Comment: Unfortunately, only about %30 of the people got the correct final answer. Most people used the right method, but sooner or later messed up the arithmetics of fractions or they lost count of the minuses. It is possible that they ran out of time, and if they had more time they would have gotten it right.

Warning: Even after I yelled "areas can never be negative", a couple of people gave a negative number as a final answer. Other people gave 0. This is also not possible for a genuine region. A few people got a negative answer but stated that it must be wrong, since areas are never negative. In an exam these people will get as much partial credit as anyone who did it the right way, but got the wrong answer due to a computational error. But people who would give a negative number for an area, volume, or length, problem (without commenting that it must be wrong) would *fail* the class!