Solutions to Attendance Quiz #19 for Dr. Z.'s Calc2 for Dec. 3, 2012

1. Find the area of the region that is bounded by the given curve and lies in the specific sector

$$r = \sin 2\theta \quad , \quad 0 \leq \theta \leq \pi/4 \quad .$$

Solution: The formula for the **area** bounded by a curve in polar $r = f(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ is

$$\frac{1}{2}\int_{\alpha}^{\beta} f(\theta)^2 \, d\theta \quad .$$

In this problem $\alpha = 0, \beta = \pi/4$ and $f(\theta) = \sin 2\theta$. So our area is:

$$\frac{1}{2}\int_{0}^{\pi/4} \sin^2 2\theta \, d\theta \quad .$$

So far most peole got it right, but only a few people knew how to continue.

Remember that whenever you have to integrate a **square-of-sine** or **square-of-cosine** you use

$$\sin^2 w = \frac{1-\cos 2w}{2} \quad ,$$

or

$$\cos^2 w = \frac{1+\cos 2w}{2} \quad .$$

This is applicable to $\sin^2 2\theta$ and even $\sin^2 100000\theta$, since w can be **whatever**.

In our case $w = 2\theta$, so

$$\sin^2 2\theta = \frac{1-\cos 4\theta}{2} \quad .$$

So our area is

$$\frac{1}{2}\int_{0}^{\pi/4} \sin^2 2\theta \, d\theta = \frac{1}{2}\int_{0}^{\pi/4} \frac{1-\cos 4\theta}{2} d\theta = \frac{1}{4}\int_{0}^{\pi/4} (1-\cos 4\theta) d\theta =$$

$$= \frac{1}{4}\left(\theta - \frac{\sin 4\theta}{4}\right)\Big|_0^{\pi/4} = \frac{1}{4}\left((\pi/4 - 0) - (\sin(0) - \sin(\pi))/4\right) = \frac{\pi}{16} \quad .$$

Ans.: The area of our region is $\frac{\pi}{16}$ square-units.