1. Find the area of the region that is bounded by the given curve and lies in the specific sector

\[ r = \sin 2\theta \quad , \quad 0 \leq \theta \leq \pi/4 \n\]

**Solution:** The formula for the area bounded by a curve in polar \( r = f(\theta) \) between \( \theta = \alpha \) and \( \theta = \beta \) is

\[
\frac{1}{2} \int_{\alpha}^{\beta} f(\theta)^2 \, d\theta .
\]

In this problem \( \alpha = 0, \beta = \pi/4 \) and \( f(\theta) = \sin 2\theta \). So our area is:

\[
\frac{1}{2} \int_{0}^{\pi/4} \sin^2 2\theta \, d\theta .
\]

So far most people got it right, but only a few people knew how to continue.

Remember that whenever you have to integrate a square-of-sine or square-of-cosine you use

\[
\sin^2 w = \frac{1 - \cos 2w}{2} ,
\]

or

\[
\cos^2 w = \frac{1 + \cos 2w}{2} .
\]

This is applicable to \( \sin^2 2\theta \) and even \( \sin^2 10000\theta \), since \( w \) can be whatever.

In our case \( w = 2\theta \), so

\[
\sin^2 2\theta = \frac{1 - \cos 4\theta}{2} .
\]

So our area is

\[
\frac{1}{2} \left[ \int_{0}^{\pi/4} \sin^2 2\theta \, d\theta \right] = \frac{1}{2} \left[ \int_{0}^{\pi/4} (1 - \cos 4\theta) \, d\theta \right] = \frac{1}{4} \left[ (\pi/4 - 0) - (\sin(0) - \sin(\pi))/4 \right] = \frac{\pi}{16} .
\]

**Ans.:** The area of our region is \( \frac{\pi}{16} \) square-units.