1. Find the radius of convergence and interval of convergence of the series

\[ \sum_{n=1}^{\infty} \frac{n^2(x-2)^n}{5^{n+2}}. \]

**Sol. of 1:** We first apply the ratio test.

\[
a_n = \frac{n^2(x-2)^n}{5^{n+2}} \quad \text{and} \quad a_{n+1} = \frac{(n+1)^2(x-2)^{n+1}}{5^{n+3}}.
\]

\[
a_{n+1} = \frac{(n+1)^2(x-2)^{n+1}}{5^{n+3}n^2} \quad \text{and} \quad a_n = \frac{n^2(x-2)^n}{5^{n+2}}.
\]

\[
\rho = \lim_{n \to \infty} \frac{(n+1)^2(x-2)^{n+1}}{5^{n+3}n^2} \cdot \frac{5^{n+2}}{n^2(x-2)^n} = \frac{(x-2)(n+1)^2}{5n^2}.
\]

We now take the limit of \( \frac{a_{n+1}}{a_n} \).

\[
\rho = \lim_{n \to \infty} \frac{(x-2)(n+1)^2}{5n^2} = \lim_{n \to \infty} \frac{(x-2)(n+1)^2}{5n^2} = \frac{x-2}{5}.
\]

We now solve \( |\rho| < 1 \).

\[
|x-2| < 5.
\]

So the radius of convergence is 5 and the center of the interval of convergence is \( x = 2 \). So the tentative interval of convergence is the open interval \(-3 < x < 7\).

We now have to test the end-points. When \( x = -3 \) our power series is:

\[
\sum_{n=1}^{\infty} \frac{n^2(-3-2)^n}{5^{n+2}} = \sum_{n=1}^{\infty} \frac{n^2(-5)^n}{5^{n+2}} = \frac{1}{25} \sum_{n=1}^{\infty} n^2(-1)^n.
\]

This series is divergent by the divergence test.

When \( x = 7 \) our series is:

\[
\sum_{n=1}^{\infty} \frac{n^2(7-2)^n}{5^{n+2}} = \sum_{n=1}^{\infty} \frac{n^25^n}{5^{n+2}} = \frac{1}{25} \sum_{n=1}^{\infty} n^2.
\]

This infinite series diverges because of the divergence test (or the \( p \)-test with \( p = -2 \) that is \( \leq 1 \)).

So neither endpoints qualifies, and the tentative interval of convergence is the final one.

**Ans. to 1:** The radius of convergence is 5 and the interval of convergence is \((-3, 7)\).