Solutions to Attendance Quiz #15 for Dr. Z.'s Calc2 for Nov. 12, 2012

1. Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^2(x-2)^n}{5^{n+2}} \quad .$$

Sol. of 1: We first apply the *ratio test.*

$$a_n = \frac{n^2(x-2)^n}{5^{n+2}}$$

$$a_{n+1} = \frac{(n+1)^2(x-2)^{n+1}}{5^{n+3}}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^2(x-2)^{n+1}}{5^{n+3}}}{\frac{n^2(x-2)^n}{5^{n+2}}}$$

$$= \frac{(n+1)^2(x-2)^{n+1}5^{n+2}}{5^{n+3}n^2(x-2)^n} = \frac{(x-2)(n+1)^2}{5n^2}$$

We now take the *limit* of $\frac{a_{n+1}}{a_n}$.

$$\rho = \lim_{n\to\infty} \frac{(x-2)(n+1)^2}{5n^2} = \lim_{n\to\infty} \frac{(x-2)(n)^2}{5n^2} = \frac{x-2}{5} \quad .$$

We now solve $|\rho| < 1$.

$$|\frac{x-2}{5}| < 1$$

Is the same as

$$|x-2| < 5 \quad .$$

So the *radius of convergence* is 5 and the *center of the interval of convergence* is $x = 2$. So the **tentative** interval of convergence is the open interval $-3 < x < 7$.

We now have to test the end-points. When $x = -3$ our power series is:

$$\sum_{n=1}^{\infty} \frac{n^2(-3-2)^n}{5^{n+2}} = \sum_{n=1}^{\infty} \frac{n^2(-5)^n}{5^{n+2}} = \frac{1}{25}\sum_{n=1}^{\infty} n^2(-1)^n$$

This series is **divergent** by the *divergence test.*

When $x = 7$ our series is:

$$\sum_{n=1}^{\infty} \frac{n^2(7-2)^n}{5^{n+2}} = \sum_{n=1}^{\infty} \frac{n^2 5^n}{5^{n+2}} = \frac{1}{25}\sum_{n=1}^{\infty} n^2 \quad .$$

This infinite series diverges because of the divergence test (or the p- test with $p = -2$ that is ≤ 1).

So neither endpoints qualifies, and the tentative interval of convergence is the final one.

Ans. to 1: The radius of convergence is 5 and the interval of convergence is $(-3, 7)$.