Solutions to the Attendance Quiz #13 for Dr. Z.'s Calc2 for Oct. 29, 2012

1. Determine whether the following series converge or diverge

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1} \quad , \quad (b) \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+2n^2} \quad , \quad (c) \sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}} \quad .$$

Solution of 1(a): (a) converges for two good reasons. First the corresponing series of absolute values, $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges by the **limit comparison test** and the p-test (in the long-run $\frac{1}{n^2+1}$ is essentially $\frac{1}{n^2}$ so the convergence status of $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is the same as that of the simpler series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and the latter is convergent by the p-test, since $p = 2$ is bigger than 1. So (a) is **absolutely convergent**, and hence convergent.

But we also know that it is convergent by the **ping-pong** test, $\frac{1}{n^2+1}$ is (i) decreasing (ii) goes to 0 as $n \to \infty$, so the series, being **alternating** is convergent.

Solution of 1(b): The limit of $\frac{(-1)^n n^2}{1+2n^2}$ does not exist (in the long-run the *sequence* is very close to $(-1)^n \frac{1}{2}$ that does not converge, so by the *divergence test* the series if **divergent**.

Remark: Even the positive version, $\sum_{n=1}^{\infty} \frac{n^2}{1+2n^2}$ is divergent. Now the limit of the **sequence** $\{\frac{n^2}{1+2n^2}\}$ does exist, but it is **non-zero**, so once again by the divergence test the series diverges.

Solution of 1(c): The sequence $\{\frac{1}{\sqrt{n}}\}$ is (i) decreasing (ii) goes to 0 as $n \to \infty$, hence by the **ping-pong test** the series, being **alternating** is **convergent**.

Remark: In many problems they ask whether the series is absolutely convergent, conditionally convergent, or divergent. If this would have been the question, then the anser is "condtionally convergent", since the positive version is a p-series with $p = \frac{1}{2}$ so it diverges.