1. Use the integral test to determine whether the series is convergent or divergent.

\[ \sum_{n=1}^{\infty} ne^{-n^2} \]

**Sol. of 1:** We consider the analogous improper integral:

\[ \int_1^{\infty} xe^{-x^2} \, dx \]

We need to do a substitution \( u = x^2 \). We have \( \frac{du}{dx} = 2x \) so \( dx = \frac{du}{2x} \). Also \( x = 1 \) goes to \( u = 1 \) and \( x = \infty \) goes to \( u = \infty \). So our improper integral is

\[
\int_1^{\infty} xe^{-u} \cdot \left( \frac{du}{2x} \right) = \frac{1}{2} \int_1^{\infty} e^{-u} \, du =
\]

\[
= \frac{1}{2} \left( e^{-u} \right|_1^{\infty} = -\frac{1}{2} \left( e^{-\infty} - e^{-1} \right) = \frac{1}{2e} .
\]

Since we got a finite answer, the improper integral (analogous to the series) is convergent, and hence by the integral test the infinite series is convergent.

**Ans. to 1:** Convergent by the integral test.

**Comments:** Only about 20% of the 8:40am class got it right completely, but 50% of the 12:00 got it right (but I did a similar problem in class). Some people had trouble with the change of variable, and got (incorrectly) that the improper integral diverges.

2. Determine whether the series are convergent or divergent

\[ (a) \sum_{n=1}^{\infty} \frac{5}{n^{.97}} \]

\[ (b) \sum_{n=1}^{\infty} \frac{2}{n^{1.001}} \]

**Sol. of 2:** You can always take numbers in front of the \( \sum \), and it does not change the convergence status. (a) is a \( p \)-series with \( p = 0.97 \), and since this is \( \leq 1 \) it diverges. (b) is also a \( p \)-series, but with \( p = 1.001 \). This is \( > 1 \) so the series converges.

**Ans. to 2:** (a) is divergent and (b) is convergent (both because of the \( p \)-test).

**Comments:** Almost everyone got it right.