Solutions to Attendance Quiz #12 for Dr. Z.'s Calc2 for Oct. 25, 2012

1. Use the integral test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} ne^{-n^2}$$

Sol. of 1: We consider the *analogous* **improper integral**:

$$\int_1^{\infty} xe^{-x^2}\, dx \quad .$$

We need to do a **substitution** $u = x^2$. We have $\frac{du}{dx} = 2x$ so $dx = \frac{du}{2x}$. Also $x = 1$ goes to $u = 1$ and $x = \infty$ goes to $u = \infty$. So our improper interal is

$$\int_1^{\infty} xe^{-u} \cdot (\frac{du}{2x}) = \frac{1}{2}\int_1^{\infty} e^{-u}\, du =$$

$$= \frac{1}{2}\frac{e{-u}}{-1}\Big|_1^{\infty} = -\frac{1}{2}(e^{-\infty} - e^{-1}) = \frac{1}{2e} \quad .$$

Since we got a **finite answer**, the *improper integral* (analogous to the series) is **convergent**, and hence by the **integral test** the infinite series is **convergent**.

Ans. to 1: Convergent by the integral test.

Comments: Only about %20 of the 8:40am class got it right completely, but %50 of the 12:00 got it right (but I did a similar problem in class). Some people had trouble with the change of variable, and got (incorrectly) that the improper integral diverges.

2. Determine whether the series are convergent or divergent

$$(a) \sum_{n=1}^{\infty} \frac{5}{n^{.97}} \quad .$$

$$(b) \sum_{n=1}^{\infty} \frac{2}{n^{1.001}} \quad .$$

Sol. of 2: You can always take numbers in front of the $\sum$, and it does not change the convergence status. (a) is a p-series with $p = 0.97$, and since this is ≤ 1 it **diverges**. (b) is also a p-series, but with $p = 1.001$. This is > 1 so the series **converges**.

Ans. to 2: (a) is divergent and (b) is convergent (both because of the p-test).

Comments: Almost everyone got it right.