1. Determine whether the sequence 
\[ a_n = \frac{n + 3}{10n + 1} \],

**Sol. of 1:** When \( n \) is very large \( a_n \) is very close to \( a_n = \frac{n}{10n} = \frac{1}{10} \) so in the limit as \( n \to \infty \) it goes to \( \frac{1}{10} \).

**Ans. to 1:** The sequence is convergent and its limit is \( \frac{1}{10} \).

2. Determine whether the sequence \( a_n = \frac{(-1)^n(3 + 5n^2)}{n + n^2} \), converges or diverges. If it converges find its limit.

**Sol. of 2:** \( a_n = \frac{(-1)^n(3 + 5n^2)}{n + n^2} \) is essentially \( a_n = \frac{(-1)^n(5n^2)}{n^2} \) that equals, thanks to algebra to \( a_n = (-1)^5 \), so in the long-run, the sequence is very close to 5, -5, 5, -5, . . . , that can’t make-up its mind between 5 and -5 so the sequence is **divergent**.

**Ans. to 2:** The sequence is divergent.

3. Determine whether the series is convergent, and if it is, find its sum.

\[ \sum_{n=1}^{\infty} \frac{1 + 2^n}{3^n} \],

**Sol. of 3:** You can break-it up to a sum of two (convergent!) geometric series:

\[ \sum_{n=1}^{\infty} \frac{1 + 2^n}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \frac{2^n}{3^n} \].

The first series:

\[ \sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{3} + \left( \frac{1}{3} \right)^2 + \left( \frac{1}{3} \right)^3 + \left( \frac{1}{3} \right)^4 + \ldots \]

is a geometric series with \( a = \frac{1}{3} \) and \( r = \frac{1}{3} \). Since \( |r| < 1 \) it is convergent and its value equals \( a/(1-r) = \frac{1}{3}/(1 - \frac{1}{3}) = \frac{1}{2} \).

The first series:

\[ \sum_{n=1}^{\infty} \frac{2^n}{3^n} = \frac{2}{3} + \left( \frac{2}{3} \right)^2 + \left( \frac{2}{3} \right)^3 + \left( \frac{2}{3} \right)^4 + \ldots \]

is a geometric series with \( a = \frac{2}{3} \) and \( r = \frac{2}{3} \). Since \( |r| < 1 \) it is convergent and its value equals \( a/(1-r) = \frac{2}{3}/(1 - \frac{2}{3}) = \frac{2}{1} = 2 \).

So the original series is a sum of two convergent series, and its value is their sums, namely \( \frac{1}{2} + 2 = \frac{5}{2} \).

**Ans. to 3:** The series is convergent, and its value equals \( \frac{5}{2} \).