Solutions to the Attendance Quiz #11 for Dr. Z.'s Calc2 for Oct. 22 , 2012

1. Determine whether the sequence

$$a_n = \frac{n+3}{10n+1} \quad ,$$

Sol. of 1: When n is very large $a_n = \frac{n+3}{10n+1}$ is very close to $a_n = \frac{n}{10n} = \frac{1}{10}$ so in the *limit* as $n \to \infty$ it goes to $\frac{1}{10}$.

Ans. to 1: The **sequence** is convergent and its limit is $\frac{1}{10}$.

2. Determine whether the sequence

$$a_n = \frac{(-1)^n(3+5n^2)}{n+n^2} \quad ,$$

converges or diverges. If it converges find its limit.

Sol. of 2: $a_n = \frac{(-1)^n(3+5n^2)}{n+n^2}$ is essentially $a_n = \frac{(-1)^n(5n^2)}{n^2}$ that equals, thanks to algebra to $a_n = (-1)^n 5$, so in the long-run, the sequence is very close to $5, -5, 5, -5, \ldots$, that can't make-up its mind bewteeen 5 and -5 so the sequence is **divergent**.

Ans. to 2: The **sequence** is divergent.

3. Determine whether the series is convergent, and if it is, find its sum.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} \quad ,$$

Sol. of 3: You can break-it up to a *sum* of two (convergent!) geometric series:

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \frac{2^n}{3^n} \quad .$$

The first series:

$$\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{3} + (\frac{1}{3})^2 + (\frac{1}{3})^3 + (\frac{1}{3})^4 + \ldots$$

is a geometric series with $a = \frac{1}{3}$ and $r = \frac{1}{3}$. Since $|r| < 1$ it is convergent and its value equals $a/(1-r) = \frac{1}{3}/(1-\frac{1}{3}) = \frac{1}{3}/(\frac{2}{3}) = \frac{1}{2}$.

The first series:

$$\sum_{n=1}^{\infty} \frac{2^n}{3^n} = \frac{2}{3} + (\frac{2}{3})^2 + (\frac{2}{3})^3 + (\frac{2}{3})^4 + \ldots$$

is a geometric series with $a = \frac{2}{3}$ and $r = \frac{2}{3}$. Since $|r| < 1$ it is convergent and its value equals $a/(1-r) = \frac{2}{3}/(1-\frac{2}{3}) = \frac{2}{3}/(\frac{1}{3}) = 2$.

So the original series is a sum of two convergent series, and its value is their sums, namely $\frac{1}{2} + 2 = \frac{5}{2}$.

Ans. to 3: The **series** is convergent, and its value equals $\frac{5}{2}$.