Solutions to the Attendance Quiz #10 for Dr. Z.'s Calc2 for Oct. 15, 2012

1. Find the fifth Taylor polynomial for $f(x) = \sin x + \cos x$ centered at $a = \pi/2$.

Sol. of 1:

We first take the first five derivatives:

$$f(x) = \sin x + \cos x$$
$$f'(x) = \cos x - \sin x \quad ,$$
$$f''(x) = -\sin x - \cos x \quad ,$$
$$f'''(x) = -\cos x + \sin x \quad ,$$
$$f^{(4)}(x) = \sin x + \cos x \quad ,$$
$$f^{(5)}(x) = \cos x - \sin x \quad ,$$

We now plug-in $x = \frac{\pi}{2}$:

$$f(\frac{\pi}{2}) = \sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1 + 0 = 1 \quad ,$$
$$f'(\frac{\pi}{2}) = \cos\frac{\pi}{2} - \sin\frac{\pi}{2} = 0 - 1 = -1 \quad ,$$
$$f''(\frac{\pi}{2}) = -\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = -1 - 0 = -1 \quad ,$$
$$f'''(\frac{\pi}{2}) = -\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = -0 + 1 = 1 \quad ,$$
$$f^{(4)}(\frac{\pi}{2}) = \sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1 + 0 = 1 \quad ,$$
$$f^{(5)}(\frac{\pi}{2}) = \cos\frac{\pi}{2} - \sin\frac{\pi}{2} = 0 - 1 = -1 \quad .$$

So the fifth-Taylor polynomial centered at $a = \frac{\pi}{2}$ is

$$T_5(x) = f(\frac{\pi}{2}) + \frac{f'(\frac{\pi}{2})}{1!}(x-\frac{\pi}{2}) + \frac{f''(\frac{\pi}{2})}{2!}(x-\frac{\pi}{2})^2 + \frac{f'''(\frac{\pi}{2})}{3!}(x-\frac{\pi}{2})^3 + \frac{f^{(4)}(\frac{\pi}{2})}{4!}(x-\frac{\pi}{2})^4 + \frac{f^{(5)}(\frac{\pi}{2})}{5!}(x-\frac{\pi}{2})^5$$
$$= 1 + \frac{(-1)}{1!}(x - \frac{\pi}{2}) + \frac{(-1)}{2!}(x - \frac{\pi}{2})^2 + \frac{1}{3!}(x - \frac{\pi}{2})^3 + \frac{1}{4!}(x - \frac{\pi}{2})^4 + \frac{(-1)}{5!}(x - \frac{\pi}{2})^5$$

$$= 1 - (x - \frac{\pi}{2}) - \frac{1}{2}(x - \frac{\pi}{2})^2 + \frac{1}{6}(x - \frac{\pi}{2})^3 + \frac{1}{24}(x - \frac{\pi}{2})^4 - \frac{1}{120}(x - \frac{\pi}{2})^5 \quad .$$

Ans. to 1:

$$1 - (x - \frac{\pi}{2}) - \frac{1}{2}(x - \frac{\pi}{2})^2 + \frac{1}{6}(x - \frac{\pi}{2})^3 + \frac{1}{24}(x - \frac{\pi}{2})^4 - \frac{1}{120}(x - \frac{\pi}{2})^5 \quad .$$

Comments: 1. About %75 of the students got it completely right. Some people got it right but left 4!, 5! as is, rather than 24 and 120 respectively. Some people forgot that the powers are of $(x - \frac{\pi}{2})$ **not** of x, since $a = \frac{\pi}{2}$ and not $a = 0$. Some people left it in terms of $\sin\frac{\pi}{2}$ and $\cos\frac{\pi}{2}$, i.e. they forgot that they equal 1 and 0 respectively.