NAME: (print!) _____

Section: _____ **E-Mail address:** _____

MATH 152 (01-03), Dr. Z. , First Midterm, Thurs. Oct. 18, 2012.

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question) (when applicable)
Explain your work! Do not write below this line

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1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)

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tot. (out of 100)

1. (10 points [5 each]) Find the following indefinite integrals

(a)

$$\int x^2 e^x \, dx \quad ;$$

Ans. to (a):

(b)

$$\int \frac{1}{x(x-1)^2} \, dx \quad .$$

Ans. to (b):

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2. (10 points) The base of a solid is the region inside the ellipse $4x^2 + y^2 = 4$. Each cross section of the solid perpendicular to the x-axis is a triangle whose height is ten times the length of the base. What is the volume of the solid?

Ans.

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3. (10 points, 5 each) Consider the region lying above the x-axis, below the curve $y = x^3$ and between the vertical lines $x = 0$ and $x = 1$. Find the volume formed by rotating it about the (a) x-axis (b) y-axis .

Ans. to (a): **Ans. to (b):**

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4. (10 pts) Find the area bounded between the curves $y = 16 - x^4$ and $y = x^4 - 16$.

Ans.

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5. (10 points, 5 each) Determine whether each of the following integrals is convergent or divergent. Evaluate those that are convergent. Be sure to explain everything.

(a)

$$\int_1^\infty \frac{1}{x^4}\,dx$$

Ans to (a).

(b)

$$\int_{10}^\infty \frac{x^{199}+x^{76}+1}{x^{200}-x^{76}+7}\,dx$$

Ans to (b).

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6. (10 pts) Find the average value of the function $f(x) = \cos^2 x$ on the interval $0 \leq x \leq \pi$. Is it larger or smaller than the average of the maximum and minimum of $\cos^2 x$ on that interval?

Answers: Average= Max= Min=

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7. (10 pts [6 for (a) and 4 for (b)]) Let

$$I = \int_2^6 \frac{3}{x} \quad .$$

Reminders:

$$S_N = \frac{1}{3}\Delta x \left[y_0 + 4y_1 + 2y_2 + \ldots + 4y_{N-3} + 2y_{N-1} + 4y_{N-1} + y_N\right] \quad ,$$

where $\Delta x = \frac{b-a}{N}$, and $y_j = f(a + j\Delta x)$. Also recall

$$Error(S_N) \leq \frac{K_4(b-a)^5}{180N^4} \quad ,$$

where K_4 is a number that that $|f^{(4)}(x)| \leq K_4$ for all $x \in [a, b]$.

(a) Use Simpson's rule with $N = 4$ subdivisions to find an approximation, call it J.

Ans to (a)

(b) Use the error estimate to find an upper bound for the error $|I - J|$.

Ans to (b)

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8. (10 points, 5 each) (a) Evaluate

$$\int x \tan^{-1} x \, dx \quad .$$

Ans to (a)

(b) Evaluate

$$\int (\ln x)^2 \, dx \quad .$$

Ans to (b)

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9. (10 pts, 5 each) (a) Evaluate

$$\int \sqrt{16 - x^2}\, dx \quad .$$

Ans to (a)

(b) Evaluate

$$\int \sin^5 x \cos^3 x\, dx \quad .$$

Ans to (b)

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10. (10 points) Evaluate

$$\int \sin^5 dx \quad ,$$

using the reduction formula

$$\int \sin^n x\, dx \;=\; -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2}(x)\, dx \quad .$$

Ans.
