NAME: (print!) _____

Section: _____ E-Mail address: _____

MATH 152 (01-03), Dr. Z., First Midterm, Thurs. Oct. 18, 2012.

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDI-CATED PLACE (right under the question) (when applicable) Explain your work! Do not write below this line

- 1. (out of 10)
- 2. (out of 10)
- 3. (out of 10)
- 4. (out of 10)
- 5. (out of 10)
- 6. (out of 10)
- 7. (out of 10)
- 8. (out of 10)
- 9. (out of 10)
- 10. (out of 10)

tot.

(out of 100)

1. (10 points [5 each]) Find the following indefinite integrals
(a)

$$\int x^2 e^x \, dx \quad ;$$

Ans. to (a):

(b)

$$\int \frac{1}{x(x-1)^2} \, dx$$

.

Ans. to (b):

2. (10 points) The base of a solid is the region inside the ellipse $4x^2 + y^2 = 4$. Each cross section of the solid perpendicular to the *x*-axis is a triangle whose height is ten times the length of the base. What is the volume of the solid?

Ans.

3. (10 points, 5 each) Consider the region lying above the x-axis, below the curve $y = x^3$ and between the vertical lines x = 0 and x = 1. Find the volume formed by rotating it about the (a) x-axis (b) y-axis.

Ans. to (a):

Ans. to (b):

4. (10 pts) Find the area bounded between the curves $y = 16 - x^4$ and $y = x^4 - 16$.

Ans.

5. (10 points, 5 each) Determine whether each of the following integrals is convergent or divergent. Evaluate those that are convergent. Be sure to explain everything.
(a)

$$\int_{1}^{\infty} \frac{1}{x^4} \, dx$$

Ans to (a).

(b)

$$\int_{10}^\infty \frac{x^{199}+x^{76}+1}{x^{200}-x^{76}+7}\,dx$$

Ans to (b).

6. (10 pts) Find the average value of the function $f(x) = \cos^2 x$ on the interval $0 \le x \le \pi$. Is it larger or smaller than the average of the maximum and minimum of $\cos^2 x$ on that interval?

Answers: Average= Max= Min=

7. (10 pts [6 for (a) and 4 for (b)]) Let

$$I = \int_2^6 \frac{3}{x} \quad .$$

Reminders:

$$S_N = \frac{1}{3}\Delta x \left[y_0 + 4y_1 + 2y_2 + \ldots + 4y_{N-3} + 2y_{N-1} + 4y_{N-1} + y_N \right]$$

where $\Delta x = \frac{b-a}{N}$, and $y_j = f(a+j\Delta x)$. Also recall

$$Error(S_N) \leq \frac{K_4(b-a)^5}{180N^4} \quad .$$

where K_4 is a number that that $|f^{(4)}(x)| \leq K_4$ for all $x \in [a, b]$.

(a) Use Simpson's rule with N = 4 subdivisions to find an approximation, call it J.

Ans to (a)

(b) Use the error estimate to find an upper bound for the error |I - J|.

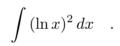
Ans to (b)

8. (10 points, 5 each) (a) Evaluate

$$\int x \tan^{-1} x \, dx \quad .$$

Ans to (a)

(b) Evaluate



Ans to (b)

9. (10 pts, 5 each) (a) Evaluate

$$\int \sqrt{16 - x^2} \, dx \quad .$$

Ans to (a)

(b) Evaluate

 $\int \sin^5 x \cos^3 x \, dx \quad .$

Ans to (b)

10. (10 points) Evaluate

$$\int \sin^5 dx \quad ,$$

using the reduction formula

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2}(x) \, dx \quad .$$

Ans.