NAME: (print!) ____________________________

Section: _____     E-Mail address: ____________

MATH 152 (01-03, 07-09), Dr. Z , Fifth Practice Exam for First Midterm Exam

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question) (when applicable)

Explain your work! Do not write below this line

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1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)

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tot.     (out of 100)
1. (10 pts, 5 each) (a) Evaluate

\[ \int \sqrt{100 - x^2} \, dx \]

(b) Evaluate

\[ \int \sin^9 x \cos^3 x \, dx \]

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Ans to (a)

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Ans to (b)
2. (10 points) Evaluate
\[ \int \sin^4 dx \]
using the reduction formula
\[ \int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} (x) \, dx \]

Ans.
3. (10 points [5 each]) Evaluate the following definite integrals:
(a) \[
\int_{0}^{1} (x - 2)e^{-3x} \, dx
\]

Ans. to (a):

(b) \[
\int_{1}^{2} \frac{10x^4 - 2}{x^5 - x} \, dx
\]

Ans. to (b):
4. (10 points) The base of a solid is the region inside the circle \( x^2 + y^2 = 100 \). Each cross section of the solid perpendicular to the \( x \)-axis is an equilateral triangle. What is the volume of the solid?

**Ans.**
5. (10 points) Find the area enclosed by the line $y = 2$ and the graph $y = \sec^2 x$ for $\frac{-\pi}{2} < x < \frac{\pi}{2}$.

Ans.
6. (10 points) Let $M$ be the average value of $f(x) = x^4$ on $[0, 3]$. Find a value of $c$ in $[0, 3]$ such that $f(c) = M$. 

Ans.
7. (10 points) Find the volume of the solid obtained by rotating about the y-axis the region enclosed by the graphs $x = \sqrt{\sin y}$, $x = 0$, $0 \leq y \leq \pi$. 

Ans.
8. (10 points) Evaluate the integral

\[ \int \frac{2x^3 + x^2 + 2x + 4}{(x^2 + 1)(x^2 + 4)} \, dx \]

Ans.
9. (10 pts [6 for (a) and 4 for (b)]) Let

\[ I = \int_{1}^{3} \frac{1}{x^3} \, dx \; ; \]

(Reminders: \( M_N = \Delta x \left[ f(c_1) + f(c_2) + \ldots + f(c_N) \right] \),
where \( \Delta x = \frac{b-a}{N} \), and \( c_j = f(a + (j-1/2)\Delta x) \). Also recall

\[ \text{Error}(M_N) \leq \frac{K^2(b-a)^3}{24N^2} \],

where \( K_2 \) is a number that that \(|f''(x)| \leq K_2\) for all \( x \in [a, b] \).)

(a) Use the midpoint rule with \( N = 2 \) subdivisions to find an approximation, call it \( J \).

Ans to (a)

(b) Use the error estimate to find an upper bound for the error \(|I - J|\).

Ans to (b)
10. (10 points) Show that \( \int_0^1 \frac{e^x}{x} \, dx \) diverges and \( \int_1^\infty xe^{-x^4} \, dx \) converges.

\[ \text{Ans.} \]