NAME: (print!) _____

Section: _____ **E-Mail address:** _____

MATH 152 (01-03, 07-09), Dr. Z. , Fourth Practice Exam for the First Midterm Exam

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question) (when applicable)
Explain your work! Do not write below this line

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1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)

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tot. (out of 100)

1. (10 pts [6 for (a) and 4 for (b)]) Let

$$I = \int_1^5 \frac{1}{x^2} \quad .$$

Reminders:

$$S_N = \frac{1}{3}\Delta x \left[y_0 + 4y_1 + 2y_2 + \ldots + 4y_{N-3} + 2y_{N-1} + 4y_{N-1} + y_N\right] \quad ,$$

where $\Delta x = \frac{b-a}{N}$, and $y_j = f(a + j\Delta x)$. Also recall

$$Error(S_N) \leq \frac{K_4(b-a)^5}{180N^4} \quad ,$$

where K_4 is a number that that $|f^{(4)}(x)| \leq K_4$ for all $x \in [a, b]$.

(a) Use Simpson's rule with $N = 4$ subdivisions to find an approximation, call it J.

Ans to (a)

(b) Use the error estimate to find an upper bound for the error $|I - J|$.

Ans to (b)

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2. (10 points, 5 each) (a) Evaluate

$$\int x^2 \tan^{-1} x \, dx \quad .$$

Ans to (a)

(b) Evaluate

$$\int (\ln x)^3 \, dx \quad .$$

Ans to (b)

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3. (10 points, 5 each) Consider the region lying above the x-axis, below the line $y = 2x$ and between the vertical lines $x = 1$ and $x = 3$. Find the area of the surface formed by rotating it about

(a) the horizontal line $y = -1$

(b) the vertical line $x = -1$.

Ans. to (a): **Ans. to (b):**

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4. (10 pts) Find the area bounded between the curve $y = 4x^3 - 1$ and the line $y = 4x - 1$.

Ans.

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5. (10 pts) Evaluate

$$\int \csc^6 x\, dx \quad ,$$

using the reduction formula

$$\int csc^m x\, dx = -\frac{\cot x \csc^{m-2} x}{m-1} + \frac{m-2}{m-1} \int \csc^{m-2} x\, dx$$

Ans.

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6. (10 pts, 5 each) Evaluate the indefinite integrals

$$(a) \int \sqrt{x} e^{\sqrt{x}} \quad , \quad (b) \int \frac{\ln(\ln x) \ln x}{x} \, dx \quad .$$

Answers (a)

(b)

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7. (10 pts altogether, 5 each) Decided whether the following improper integrals are convergent or divergent. In the former case, evaluate them.

$$(a)\int_1^\infty \frac{1+x}{x^3}\,dx \quad (b)\int_0^1 \frac{2x^{1/3}+3x^{1/4}}{x}\,dx$$

Answers (a)

(b)

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8. (10 pts) Find the average of the function $f(x) = x \ln x$ in the interval $[1, e]$

Ans.

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9. (10 pts, 5 each) Evaluate

$$(a) \int \tan(3x)\sec^2(3x)\,dx \quad , \quad (b) \int s\cos^3(s^2)\,ds \quad .$$

Answers: (a)

(b)

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10. (10 pts [6 for (a) and 4 for (b)]) Let

$$I = \int_1^4 \frac{1}{x}\, dx \quad ;$$

(**Reminders**:

$$M_N = \Delta x \left[f(c_1) + f(c_2) + \ldots + f(c_N) \right] \quad ,$$

where $\Delta x = \frac{b-a}{N}$, and $c_j = f(a + (j - 1/2)\Delta x)$. Also recall

$$Error(M_N) \leq \frac{K_2(b-a)^3}{24N^2} \quad ,$$

where K_2 is a number that that $|f''(x)| \leq K_2$ for all $x \in [a, b]$.)

(a) Use The midpoint rule with $N = 3$ subdivisions to find an approximation, call it J.

Ans to (a)

(b) Use the error estimate to find an upper bound for the error $|I - J|$.

Ans to (b)
