NAME: (print!) _____

Section: _____ **E-Mail address:** _____

MATH 152 (01-03, 07-09), Dr. Z. , Third Practice Exam for The First Midterm

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question) (when applicable)

Explain your work! Do not write below this line

---

1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)

---

tot. (out of 100)

1. (10 pts) Evaluate the definite integral

$$\int_0^{\pi} \sin^6 \, dx \quad ,$$

using the reduction formula

$$\int \sin^n x \, dx \; = \; -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2}(x) \, dx \quad .$$

Answer:

---

2. (10 pts) Compute the volume obtained by rotating the region enclosed by the curves

$$y = \sqrt{x} \quad , \quad y = x^2 \quad ,$$

around the y-axis.

Answer

---

3. (10 pts) Compute the volume obtained by rotating the region enclosed by the curves

$$y = \sqrt{x} \quad , \quad y = x^2 \quad ,$$

around the x-axis.

Answer

---

4. (10 pts) Evaluate the indefinite integral

$$\int \frac{ds}{(s^2+4)^2} \quad .$$

Answer:

---

5. (10 points, 5 each) Determine whether each of the following integrals is convergent or divergent. Evaluate those that are convergent. Be sure to explain everything.

(a)

$$\int_2^\infty \frac{48}{x^3}\,dx$$

Ans to (a).

(b)

$$\int_{20}^\infty \frac{x^{299}+x^{76}+1}{x^{300}-x^{276}+7}\,dx$$

Ans to (b).

---

6. (10 pts) Find the average value of the function $f(x) = \cos^3 x$ on the interval $0 \leq x \leq \frac{\pi}{2}$. Is it larger or smaller than the average of the maximum and minimum of $\cos^3 x$ on that interval?

Answers: Average= Max= Min=

---

7. (10 points, 5 each) Determine whether each of the following improper integrals is convergent or divergent. Evaluate those that are convergent, if possible. Be sure to explain everything.

(a)

$$\int_0^1 x^{-\frac{2}{5}}\, dx$$

Ans to (a).

(b)

$$\int_{10}^{\infty} \frac{x^{1099} + x^{76} + 1}{x^{1103} - x^{76} + 7}\, dx$$

Ans to (b).

---

8. (10 pts) Find the average value of the function $f(x) = 1 + x^2$ on the interval $0 \leq x \leq 2$. Is it larger or smaller than the average of the maximum and minimum of $1 + x^2$ on that interval?

Answers: Average= Max= Min=

---

9. (10 pts) Find the area enclosed by the curves

$$x = y^3 - 18y \quad , \quad y + 2x = 0 \quad .$$

Answer

---

10. (10 pts) Find the average of the function $f(x) = x^2 e^x$ over the interval $[0, \ln 2]$.

Answer
