NAME: (print!) _____

Section: _____ **E-Mail address:** _____

MATH 152 (01-03, 07-09), Dr. Z. , Second Practice Exam for The First Midterm Exam

Updated Oct. 16, 2012, 10:05pm, thanks to Priya [In problem 3: I meant volume but before I said "surface area"]

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question) (when applicable)

Explain your work! Do not write below this line

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1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)

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tot. (out of 100)

1. (10 pts) Find the area bounded by the following

$$y = \cos x \quad , \quad y = \cos 2x \quad , \quad x = 0, \quad x = \frac{2\pi}{3} \quad .$$

Ans.

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2. (10 pts [6 for (a) and 4 for (b)]) Let

$$I = \int_0^2 x^2\, dx \quad ;$$

(**Reminders**:

$$M_N = \Delta x\, [f(c_1) + f(c_2) + \ldots + f(c_N)] \quad ,$$

where $\Delta x = \frac{b-a}{N}$, and $c_j = f(a + (j - 1/2)\Delta x)$. Also recall

$$Error(M_N) \leq \frac{K_2(b-a)^3}{24N^2} \quad ,$$

where K_2 is a number that that $|f''(x)| \leq K_2$ for all $x \in [a, b]$.)

(a) Use The midpoint rule with $N = 4$ subdivisions to find an approximation, call it J.

Ans to (a)

(b) Use the error estimate to find an upper bound for the error $|I - J|$.

Ans to (b)

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3. [**corrected Oct. 16, 2012, thanks to Priya Shah**] (10 points, 5 each) Consider the region lying above the x-axis, below the curve $y = x^5$ and between the vertical lines $x = 0$ and $x = 2$. Find the volume of the solid body formed by rotating it about the (a) x-axis (b) y-axis .

Ans. to (a): **Ans. to (b):**

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4. (10 pts) Find the area bounded between the curves $y = 32 - x^6$ and $y = x^6 - 32$.

Ans.

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5. (10 pts, 5 each) Decide whether the following improper integrals are convergent or divergent. Evaluate them if you can.

$$(a) \int_1^\infty \frac{x^4 + 2x - 11}{x^5 + x}$$

$$(b) \int_0^1 \frac{1}{x^3}$$

Ans. (a) (b)

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6. (10 pts) Evaluate

$$\int \frac{x^3 + x^2 + 1}{x + 3}\, dx \quad .$$

Ans.

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7. (10 pts [6 for (a) and 4 for (b)]) Let

$$I = \int_0^4 x^3\, dx \quad ;$$

(**Reminders**:

$$T_N = \frac{1}{2}\Delta x\,[y_0 + 2y_1 + \ldots + 2y_{N-1} + y_N] \quad ,$$

where $\Delta x = \frac{b-a}{N}$, and $y_j = f(a + j\Delta x)$. Also recall

$$Error(T_N) \leq \frac{K_2(b-a)^3}{12N^2} \quad ,$$

where K_2 is a number that that $|f''(x)| \leq K_2$ for all $x \in [a, b]$.)

(a) Use The trapezoind rule with $N = 4$ subdivisions to find an approximation, call it J.

Ans to (a)

(b) Use the error estimate to find an upper bound for the error $|I - J|$.

Ans to (b)

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8. (10 points, 5 each) (a) Evaluate

$$\int \sec^3 x \, dx \quad .$$

[Reminder: $\sec^2 x - \tan^2 x = 1$]

Ans to (a)

(b) Evaluate

$$\int \left(4 \ln x + x^2 + \cos^2 x\right) \, dx \quad .$$

Ans to (b)

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9. (10 pts) Evaluate

$$\int \frac{x^2 + 3x - 44}{(x+3)(x+5)(3x-2)}\, dx \quad .$$

Ans.

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10. (10 pts) **modified Oct. 11, 2012** Use the any to calculate the volume of rotation about the x-axis of the region bounded by $y = 4 - x^2$, and the x-axis.

Ans.
