Dr. Z's Calc2 Handout for Lecture 9: Numerical Integration

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Problem Type P9.1: Use (a) The Trapezoid Rule, (b) The Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with the specified value of n

$$\int_a^b f(x)\ dx \quad , \quad n = EvenInteger$$

Example Problem P9.1: Use (a) The Trapezoid Rule, (b) The Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with the specified value of n

$$\int_2^6 \frac{1}{x}\ dx \quad , \quad n = 8$$

Steps

1. Find $\Delta x = (b-a)/n$, and write down

$$x_0 = a, x_1 = a + \Delta x, x_2 = a + 2\Delta x, \ldots,$$

$$x_n = a + n\Delta x = b \ldots,$$

For the Midpoint rule, you also need the midpoints of $[x_i, x_{i+1}]$ (call it $\bar{x_i}$) (for $i = 1, \ldots, n$).

Example

1. $\Delta x = (6-2)/8 = .5$.

$$x_0 = 2, x_1 = 2.5, x_2 = 3, x_3 = 3.5, x_4 = 4,$$

$$x_5 = 4.5, x_6 = 5, x_7 = 5.5, x_8 = 6$$

$$\bar{x}_1 = 2.25, \bar{x}_2 = 2.75, \bar{x}_3 = 3.25, \bar{x}_4 = 3.75,$$

$$\bar{x}_5 = 4.25, \bar{x}_6 = 4.75, \bar{x}_7 = 5.25, \bar{x}_8 = 5.75,$$

2. For the Midpoint Rule the approximation, called M_n is

$$M_n = \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \ldots + f(\bar{x}_n)]$$

For the Trapezoid Rule the approximation, called T_n is

$$T_n = \frac{\Delta x}{2}[f(x_0)+$$

$$2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1})$$

$$+f(x_n)]$$

For Simpson's Rule the approximation, called M_n is (now n must be even!)

$$S_n = \frac{\Delta x}{3}[f(x_0)+$$

$$4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots$$

$$+2f(x_{n-2}) + 4f(x_{n-1})$$

$$+f(x_n)]$$

2.

$M_8 = (.5)[f(2.25)+f(2.75)+f(3.25)+f(3.75)+$

$f(4.25) + f(4.75) + f(5.25) + f(5.75)] =$

$(.5)[1/(2.25)+1/(2.75)+1/(3.25)+1/(3.75)+$

$1/(4.25)+1/(4.75)+1/(5.25)+1/(5.75)] = 1.0936247\ldots$

$T_8 = \frac{(.5)}{2}[f(2)+2\cdot(f(2.5)+f(3)+f(3.5)+f(4)$

$+f(4.5) + f(5) + f(5.5)) + f(6)]$

$(.25)[1/2 + 2\cdot(1/(2.5) + 1/3 + 1/3.5 + 1/4$

$+1/(4.5)+1/5+1/5.5)+1/6] = 1.10321067\ldots$

$S_8 = \frac{(.5)}{3}[f(2)+4f(2.5)+2f(3)+4f(3.5)+2f(4)+$

$4f(4.5) + 2f(5) + 4f(5.5) + f(6)]$

$= (1/6)[1/2+4/(2.5)+2/3+4/3.5+2/4+4/4.5+$

$2/5 + 4/(5.5) + 1/6] = 1.09872534\ldots$