NAME:____

MATH 152, Dr. Z. , **Practice for Second Midterm**
Due: November 28 (you must bring it, or Exam II will not count).

1. (10 points [5 each]) For each of the two series below, determine whether they converge or diverge .

$$(a)\ \sum_{n=1}^{\infty}(-3)^n \quad , \quad (b)\ \sum_{n=1}^{\infty}\frac{n^2}{5n^2+6} \quad ,$$

2. (10 points) Use the integral test to determine whether the series is convergent or divergent. Explain everything!

$$\sum_{n=1}^{\infty} ne^{-n}$$

3. (10 points, 5 each)

Determine whether the following series converge or diverge. Explain what test(s) you are using.

$$(a) \quad \sum_{n=1}^{\infty} \frac{7+4\sqrt{n}}{n^3} \quad ,$$

$$(b) \quad \sum_{n=1}^{\infty} \frac{7+4\sqrt{n}}{n^{4/3}} \quad .$$

4. Use an improper integral to find an integer N, so that the partial sum

$$S_N = \sum_{n=1}^{N} \frac{1}{n^2}$$

is within 10^{-5} of the sum of the whole infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Be sure to explain why the value of N you give is the correct answer. Do not evaluate S_N.

5. (10 points: 3,3,4 resp.) Determine whether the following series converge or diverge

$$(a)\sum_{n=1}^{\infty}\frac{\sin^2 n}{n^3+1} \quad , \quad (b)\sum_{n=1}^{\infty}\frac{1}{n-4\sqrt{n}} \quad , \quad (c)\sum_{n=1}^{\infty}\frac{1}{2-3^{-n}} \quad .$$

6. (10 points, 3,3,4, resp.) Determine whether the following series converge or diverge (a) $\sum_{n=1}^{\infty} \frac{9+11n}{(n^2+1)^2}$, (b) $\sum_{n=1}^{\infty} \frac{1+2^n}{5+3^n}$, (c) $\sum_{n=1}^{\infty} \frac{n^2+6}{(n^9+11)^{1/3}}$.

7. (10 points) Use the sum of the first 3 terms to approximate the sum of the series. Estimate the error.

$$\sum_{n=1}^{\infty} \frac{n+4}{(n+7)5^n} \quad .$$

8. (10 points, 3,3,4, resp.) Determine whether the following series converge or diverge

$$(a)\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1} \quad , \quad (b)\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n}}{1+2\sqrt{n}} \quad , \quad (c)\sum_{n=2}^{\infty}\frac{(-1)^n}{(\ln n)^2} \quad .$$

9. (10 points, 5 each) Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

$$(a) \quad \sum_{n=1}^{\infty} \frac{n^5}{(-6)^n} \quad ,$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \quad ,$$

10. (10 points, 5 each) Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

$$(a) \sum_{n=1}^{\infty} \frac{(-2)^n n!}{n^n} \quad .$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!} \quad .$$

11. (10 points) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^2(x+2)^n}{3^n} \quad .$$

12. (10 points) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n3^n} \quad .$$

13. (10 points) Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{x}{2+32x^4} \quad .$$

14. (10 points) Evaluate the indefinite integral as a power series. What is the radius of convergence?

$$\int \frac{x^5}{8 - x^3}\, dx$$

15. (10 points) Find the Maclaurin series for $f(x) = 3\cos 2x$ using the definition of a Maclaurin series.

16. (10 points) Find the Taylor series for $f(x) = \sin 2x$ centered at $a = \pi/4$.

17. (10 points) Use known Maclaurin series to obtain the Maclaurin series for $f(x) = xe^{4x}$.

18. (10 points) Find the first four non-zero terms of the Maclaurin expansion of

$$f(x) = e^{-3x}\sin(2x)$$

19. (10 points, 5 each) (a) Expand $\sqrt[10]{1+x}$ as a power series. (b) Use part (a) to estimate $\sqrt[10]{1.01}$ correct to four decimal places.

20. (10 points, 5 each) (a) Use the binomial series to expand $1/\sqrt{1+t^2}$. (b) Use part (a) to find the Maclaurin series for

$$f(x) = \int_0^x \frac{1}{\sqrt{1+t^2}}\, dt$$