NAME:______________________________

MATH 152, Dr. Z., Practice for Second Midterm
Due: November 28 (you must bring it, or Exam II will not count).

1. (10 points [5 each]) For each of the two series below, determine whether they converge or diverge.

   \[ (a) \sum_{n=1}^{\infty} (-3)^n \quad , \quad (b) \sum_{n=1}^{\infty} \frac{n^2}{5n^2 + 6} \quad . \]
2. (10 points) Use the integral test to determine whether the series is convergent or divergent. Explain everything!

$$\sum_{n=1}^{\infty} ne^{-n}$$
3. (10 points, 5 each)
Determine whether the following series converge or diverge. Explain what test(s) you are using.

\[(a) \sum_{n=1}^{\infty} \frac{7 + 4\sqrt{n}}{n^3}, \]

\[(b) \sum_{n=1}^{\infty} \frac{7 + 4\sqrt{n}}{n^{4/3}}. \]
4. Use an improper integral to find an integer $N$, so that the partial sum

$$S_N = \sum_{n=1}^{N} \frac{1}{n^2}$$

is within $10^{-5}$ of the sum of the whole infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Be sure to explain why the value of $N$ you give is the correct answer. Do not evaluate $S_N$. 
5. (10 points: 3,3,4 resp.) Determine whether the following series converge or diverge

\[(a) \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3 + 1}, \quad (b) \sum_{n=1}^{\infty} \frac{1}{n - 4 \sqrt{n}}, \quad (c) \sum_{n=1}^{\infty} \frac{1}{2 - 3^{-n}}.\]
6. (10 points, 3,3,4, resp.) Determine whether the following series converge or diverge (a) 
\[ \sum_{n=1}^{\infty} \frac{2+11n}{(n^2+1)^2}, \] (b) \[ \sum_{n=1}^{\infty} \frac{1+2^n}{3+3^n}, \] (c) \[ \sum_{n=1}^{\infty} \frac{n^2+6}{(n^3+11)^{1/3}}. \]
7. (10 points) Use the sum of the first 3 terms to approximate the sum of the series. Estimate the error.

\[ \sum_{n=1}^{\infty} \frac{n + 4}{(n + 7)^5 n} . \]
8. (10 points, 3,3,4, resp.) Determine whether the following series converge or diverge

\[(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 1}, \quad (b) \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{1 + 2\sqrt{n}}, \quad (c) \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}.\]
9. (10 points, 5 each) Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

\[(a) \quad \sum_{n=1}^{\infty} \frac{n^5}{(-6)^n},\]

\[(b) \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}},\]
10. (10 points, 5 each) Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

\[(a) \sum_{n=1}^{\infty} \frac{(-2)^nn!}{n^n} \ .\]

\[(b) \sum_{n=1}^{\infty} \frac{(-1)^nn^n}{n!} \ .\]
11. (10 points) Find the radius of convergence and interval of convergence of the series

\[ \sum_{n=1}^{\infty} \frac{n^2(x + 2)^n}{3^n} \].
12. (10 points) Find the radius of convergence and interval of convergence of the series

\[ \sum_{n=1}^{\infty} \frac{(x + 2)^n}{n3^n} . \]
13. (10 points) Find a power series representation for the function and determine the interval of convergence.

\[ f(x) = \frac{x}{2 + 32x^4} . \]
14. (10 points) Evaluate the indefinite integral as a power series. What is the radius of convergence?

\[ \int \frac{x^5}{8 - x^3} \, dx \]
15. (10 points) Find the Maclaurin series for \( f(x) = 3 \cos 2x \) using the definition of a Maclaurin series.
16. (10 points) Find the Taylor series for $f(x) = \sin 2x$ centered at $a = \pi/4$. 
17. (10 points) Use known Maclaurin series to obtain the Maclaurin series for \( f(x) = xe^{4x} \).
18. (10 points) Find the first four non-zero terms of the Maclaurin expansion of

\[ f(x) = e^{-3x} \sin(2x) \]
19. (10 points, 5 each) (a) Expand \(\sqrt{1 + x}\) as a power series. (b) Use part (a) to estimate \(\sqrt{1.01}\) correct to four decimal places.
20. (10 points, 5 each) (a) Use the binomial series to expand \(1/\sqrt{1 + t^2}\). (b) Use part (a) to find the Maclaurin series for

\[
f(x) = \int_0^x \frac{1}{\sqrt{1 + t^2}} \, dt
\]