Problem Type 9.3a: Solve the differential equation

\[ y' = \frac{A(x)}{B(y)} \quad \text{or} \quad y' = A(x)B(y) \quad \text{etc.} \]

Example Problem 9.3a: Solve the differential equation

\[ y' = y^2 \sec x \]

Steps

1. If not already written like this, replace \( y' \) by \( \frac{dy}{dx} \). Treat \( dy \) and \( dx \) as algebraic quantities and separate the \( x \) part from the \( y \) part.

\[ \frac{dy}{dx} = \frac{A(x)}{B(y)} \quad \text{or} \quad \frac{dy}{dx} = A(x)B(y) \quad \text{etc.} \]

implies

\[ B(y)dy = A(x)dx \quad \text{respectively} \]

\[ \frac{dy}{B(y)} = A(x)dx \quad \text{etc.} \]

2. Apply the Integral sign to both sides, and perform the integration. Only put the \(+C\) on one side.

\[ \int y^{-2} \, dy = \int \sec x \, dx \]

which is the same as

\[ y^{-2} \, dy = \sec x \, dx \]

implies

\[ \int \frac{dy}{y^2} = \int \sec x \, dx \]

\[ y^{-1} = \ln |\sec x + \tan x| + C \]
3. If possible, solve for \( y \). Otherwise leave it in implicit form. If there is an initial condition then plug it in and solve for \( C \). If nothing is mentioned (like in this problem), then leave \( C \) alone.

3. 
\[
y = \frac{-1}{\ln|\sec x + \tan x| + C}
\]

Ans.: 
\[
y = \frac{-1}{\ln|\sec x + \tan x| + C}.
\]

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**Problem Type 9.3b**: Find an equation of the curve that passes through the point \((a, b)\) and whose slope at \((x, y)\) is \(A(y)/B(x)\).

**Example Problem 9.3b**: Find an equation of the curve that passes through the point \((1, 1)\) and whose slope at \((x, y)\) is \(y^2/x^3\).

**Steps**

1. Slope is derivative, so set it equal to \(dy/dx\). Treat \(dy\) and \(dx\) as algebraic quantities and **separate** the \(x\) part from the \(y\) part.

\[
\frac{dy}{dx} = \frac{A(y)}{B(x)}
\]

implies

\[
\frac{dy}{A(y)} = \frac{dx}{B(x)}
\]

2. Apply the Integral sign to both sides, and perform the integration. Only put the +\(C\) on one side.

\[
y^2 \frac{dy}{y^2} = x^3 \frac{dx}{x^3}
\]

which is the same as

\[
y^{-2} \frac{dy}{x^{-3}} = dx
\]

1. \[
\frac{dy}{dx} = \frac{y^2}{x^3}
\]

implies

\[
\frac{dy}{y^2} = \frac{dx}{x^3}
\]

which is the same as

\[
y^{-2} \frac{dy}{x^{-3}} = dx
\]

2. \[
\frac{y^{-1}}{-1} = \frac{x^{-2}}{-2} + C
\]

which gives

\[
\frac{-1}{y} = \frac{-1}{2x^2} + C
\]
3. Plug in the point \((x = a, y = b)\) and solve for \(C\). Plug back that value for \(C\)
and try to express \(y\) in terms of \(x\) if possible. Otherwise leave it in implicit form.

\[
\frac{-1}{1} = \frac{-1}{2 \cdot 1^2} + C
\]
giving \(C = -1/2\). Incorporating that \(C\) gives
\[
\frac{-1}{y} = \frac{-1}{2x^2} - \frac{1}{2}
\]
and algebra gives
\[
y = \frac{2x^2}{1 + x^2}.
\]

Ans.: \(\frac{2x^2}{1 + x^2}\).