

Dr. Z's Math152 Handout #6.5 [Average Value of a Function]

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Problem Type 6.5a: Find the average value of a function $f(x)$ in the interval $[a, b]$ and find c such that $f(c) = f_{ave}$.

Example Problem 6.5a: Find the average value of $f(x) = (x - 2)^2$ in the interval $[1, 4]$ and find c such that $f(c) = f_{ave}$.

Steps

1. Use the formula

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

2. Evaluate the integral

3. Solve, for c , $f(c) = f_{ave}$. Only retain the solutions that lie in the interval $[a, b]$.

Example

1. Set up the integral

$$f_{ave} = \frac{1}{4-1} \int_1^4 (x-2)^2 dx$$

- 2.

$$\begin{aligned} f_{ave} &= \frac{1}{4-1} \int_1^4 (x-2)^2 dx = \frac{1}{3} \int_1^4 (x-2)^2 dx \\ &= \frac{1}{3} \left(\frac{(x-2)^3}{3} \Big|_1^4 \right) = \frac{1}{3} \left(\frac{(4-2)^3}{3} - \frac{(1-2)^3}{3} \right) = 1. \end{aligned}$$

3. We have to solve $(c-2)^2 = 1$, i.e. $c-2 = \pm 1$ giving the solutions $c = 3$ and $c = 1$. In this case they both lie there, so
Ans.: $f_{ave} = 1$; $c = 1$ and $c = 3$.