Dr. Z’s Math152 Handout #6.5 [Average Value of a Function]

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**Problem Type 6.5a**: Find the average value of a function \( f(x) \) in the interval \([a, b]\) and find \( c \) such that \( f(c) = f_{\text{ave}} \).

**Example Problem 6.5a**: Find the average value of \( f(x) = (x - 2)^2 \) in the interval \([1, 4]\) and find \( c \) such that \( f(c) = f_{\text{ave}} \).

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**Steps**

1. Use the formula

\[
f_{\text{ave}} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
\]

**Example**

2. Evaluate the integral

\[
f_{\text{ave}} = \frac{1}{4 - 1} \int_{1}^{4} (x - 2)^2 \, dx
\]

\[
= \frac{1}{3} \left( \frac{(4 - 2)^3}{3} - \frac{(1 - 2)^3}{3} \right) = 1.
\]

3. Solve, for \( c \), \( f(c) = f_{\text{ave}} \). Only retain the solutions that lie in the interval \([a, b]\).

**Example**

3. We have to solve \((c - 2)^2 = 1\), i.e. \(c - 2 = \pm 1\) giving the solutions \(c = 3\) and \(c = 1\). In this case they both lie there, so

\[\text{Ans.}: f_{\text{ave}} = 1; c = 1 \text{ and } c = 3.\]