Dr. Z's Math152 Handout #6.4 [Work]

By Doron Zeilberger

Problem Type 6.4a:

A spring has natural length of a meters. If an F_0-Newton force is required to keep it stretched to b meters, how much work is needed to stretch it from c meters to d meters?

Example Problem 6.4a:

A spring has natural length of 20 cm. If an 25-Newton force is required to keep it stretched to 30 cm, how much work is needed to stretch it from 25 cm to 30 cm?

Steps

1. Find Hooke's constant k, given by Force divided by "stretched length" (w.r.t. to the natural length). (i.e. $F_0/(b-a)$) Make sure you convert everything to standard units! The formula for the force is : $F = kx$ (where x is length beyond natural length).

2. Set up the integral for the work

$$Work = \int_{c-a}^{d-a} kx \; dx$$

3. Evaluate the integral.

Example

1. The "stretched length" is $30 - 20 = 10$ cm, and in meters, $.1m$. According to the problem, the force then is 25. So $k = 25/.1 = 250$. The formula for force in general is $F = 250x$.

2. Convering 20, 25, and 30 cm to meters,

$$Work = \int_{.05}^{.1} 250x \; dx$$

3.

$$Work = \int_{.05}^{.1} 250x \; dx = 125x^2|_{.05}^{.1} = 125((.1)^2 - (.05)^2) = 15/16J.$$

Ans.: $15/16J$.

Problem Type 6.4b:

A cable that weights a lb/ft is used to lift B lb of coal up a mineshaft C ft deep. Find the work done.

Example Problem 6.4b:

A cable that weights 2 lb/ft is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work done.

Steps

1. Find the force (in this case to overcome gravity, i.e. *weight*) at depth x. We have the constant weight (B) due to the coal and the changing weight due to the cable (ax). The force $F(x)$ equals $B + ax$.

2. Set up the integral for the work

$$Work = \int_0^C F(x)\ dx$$

3. Evaluate the integral.

Example

1. $F(x) = 800 + 2x$.

2.

$$Work = \int_0^{500} (800 + 2x)\ dx$$

3.

$$Work = \int_0^{500} (800+2x)\ dx \ = \ (800x+x^2)|_0^{500}$$

$$= (800 \cdot (500) + 500^2) - 0 = 650,000 \quad .$$

Ans.: 650,000 ft-lb.